Kinematic Analysis of Rack and Pinion Steering System of Rural Multi Purpose Vehicle

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Abstract

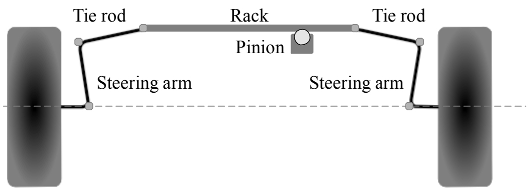
Steering linkages play an important role in maneuvering of cars. In order to provide pure rolling of the road wheels and to reduce wear of the tires, the steering linkage must be able to turn the wheels such that their axis intersection point lies on the rear wheel axis. This condition is known as the Ackermann condition. However, in reality, Ackermann condition is difficult to satisfy for every cornering radius. The only effort we can do is to synthesize the linkage so that the Ackermann condition is satisfied for any turning radius as closely as possible. Hence, an appropriate kinematic model of the steering linkage is essential. The purpose of this research is to analyze the rack and pinion steering linkage for the rural multipurpose vehicle. From this analysis, the information on the steering linkage dimension and the placement of the steering linkage that give minimum steering error can be obtained. The steering error is defined as the difference between the actual angle made by the outer front wheel during steering maneuvers and the correct angle for the same wheel based on the Ackerman principle. In addition, this analysis helps and can be used as starting point to design the chassis and cabin for the rural multi-purpose vehicle. Real steering mechanisms are complex spatial linkages and the variables defining their geometry are quite numerous. However, the kingpin inclination and caster angles that provide compliance to the steering linkage with the suspension system have little influence on the motion transmission of the steering linkage. As a result, the real rack and pinion steering linkage, which is spatial in nature, can be modelled as a planar linkage for the investigation of Ackermann condition. Therefore, by neglecting the kingpin inclination and caster angle, in this study, the steering linkage of a vehicle is considered as a planar linkage. From the analysis, it is found that placing the rack closer to wheels axis will reduce the turning radius of the vehicle. However, by placing the rack around 6 - 8 cm in front of wheel axis will give minimum steering error. The length of tie-rod and steering arm depend on the placement of the rack.

Key Words: Steering linkage, Rack and Pinion, Kinematic Analysis, Rural Multi Purpose Vehicle, Steering Error.

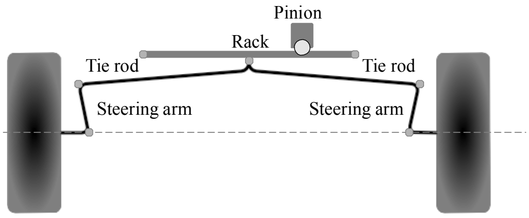
Introduction

Steering linkages play an important role in maneuvering of cars. Among the steering linkages, rack and pinion steering (RPS) linkage is the most widely used in passenger cars. The advantages of rack and pinion steering system over the others are 1) simple construction; 2) economical and uncomplicated to manufacture; 3) compact and easy to operate (Reimpell et al., 2002). The rack and pinion steering system consists of two steering arms, two tie rods, and a rack, as shown in Fig. 1. There are two different embodiments of such steering mechanisms. The standard arrangement has the tie-rods connected to the ends of the rack (Fig. 1(a)), is called the side take-off rack and pinion steering mechanism. The other with the central joints very close to each other (Fig. 1(b)) is called central take-off rack and pinion steering mechanism.

In order to provide pure rolling and to reduce wear of the tires, a steering linkage must be able to maneuver the vehicle so that it follows the Ackermann principle, see Fig. 2. This principle states that during low speed cornering and free from lateral inertia forces, the line drawn from the center of the wheels should meet at the center of bend, i.e., point O of Fig. 2. Referring to Fig. 2, the relation between the inner wheel angle, δ_i, and the outer wheel angle according to Ackerman principle, δ_{oA},



(a) Side take-off



(b) Central take-off

Fig. 1: Rack and Pinion steering system configuration

is given by

$$\delta_{oA} = \tan^{-1} \frac{1}{\cot \delta_i + \frac{W_t}{W_b}} \tag{1}$$

where W_t is wheel track and W_b is wheel base.

In reality, condition in Eq. (1) is difficult to satisfy for every cornering radius. The only effort we can do is to synthesize the linkage so that the Acker-

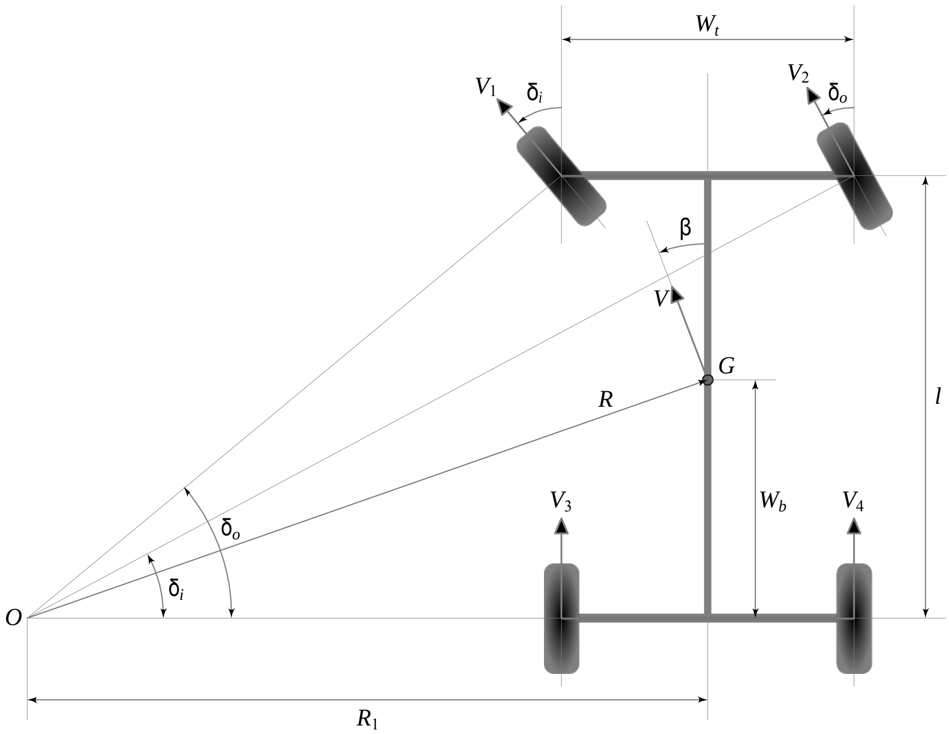


Fig. 2: Steering kinematics

mann condition is satisfied for any turning radius as closely as possible. Hence, an appropriate kinematic model of the steering linkage is essential. Steering linkages have received a lot of attention from the kinematics point of view because of its influence on the Ackermann error (Simionescu and Smith, 2000). Less attention is paid to its dynamics apparently caused by the fact that the pinion rotates at low speed, typically 10-15 rpm (Hanzaki et al., 2007).

Real steering mechanisms are complex spatial linkages. However, the kingpin inclination and caster angles that provide compliance to the steering linkage with the suspension system have little influence on the motion transmission of the steering linkage (Simionescu and Smith, 2000). As a result, the real rack and pinion steering linkage, which is spatial in nature, can be modeled as a planar linkage for the investigation of Ackermann condition. Therefore, by neglected the kingpin inclination and caster angle, in this study, the steering linkage of a vehicle is considered as a planar linkage. Such a simplification of the steering system has been also used by other researchers, e.g. (Simionescu and Smith, 2000; Hanzaki et al., 2009).

The purpose of this research is to analyze kinematically the rack and pinion steering linkage for the rural multipurpose vehicle. From this analysis, the information on the steering linkage dimension and the placement of the steering linkage that give minimum steering error can be obtained. The steering error is defined as the difference between the actual angle made by the outer front wheel during steering maneuvers and the correct angle for the same wheel based on the Ackerman principle. In addition, this analysis helps and can be used as starting point to design the chassis and cabin for the rural multi-purpose vehicle.

Kinematic Modeling

The side take-off configuration of the steering linkage, shown in Fig. 1(a), is more common in passenger cars. Let redraw the side take-off steering linkage for kinematic analysis as shown in Fig. 3(a). These linkages noted $C_l B_l A_l A_r B_r C_r$, has an infinite number of cognates Simionescu and Smith (2000), meaning that the input-output link behaviors of all the cognate linkages are same. Further, the steering linkage shown in Fig. 3(a) can be represented as shown in Fig. 3(b) which is the cognate of that shown in Fig. 3(a). Moreover, l_1 and l_4 denote the steering arms of length l_a, whereas l_2 and l_3 represent tie-rods of length l_t. The length of w_t is defined as

$$\mathsf{w}_t = W_t - L_r \tag{2}$$

where L_r is length of the rack. Define $k_l = \frac{\mathsf{w}_t}{2} + s$ and $k_l = \frac{\mathsf{w}_t}{2} - s$ with s is the rack displacement, from Fig. 3(b)

$$\begin{aligned} l_1\cos\theta_1 + l_2\cos\theta_2 &= k_l \\ l_1\sin\theta_1 + l_2\sin\theta_2 &= h \end{aligned} \tag{3}$$

$$\begin{aligned} l_4\cos\theta_4 + l_3\cos\theta_3 &= k_r \\ l_4\sin\theta_4 + l_3\sin\theta_3 &= h \end{aligned} \tag{4}$$

Eq. (3) and Eq. (4) can be written as

$$\begin{aligned} l_2\cos\theta_2 &= k_l - l_1\cos\theta_1 \\ l_2\sin\theta_2 &= h - l_1\sin\theta_1 \end{aligned} \tag{5}$$

$$\begin{aligned} l_3\cos\theta_3 &= k_r - l_4\cos\theta_4 \\ l_3\sin\theta_3 &= h - l_4\sin\theta_4 \end{aligned} \tag{6}$$

By squaring and summation in Eq. (5) we get

$$l_2^2 = k_l^2 + h^2 + l_1^2 - 2k_l l_1\cos\theta_1 - 2hl_1\sin\theta_1 \tag{7}$$

Considering $A_l = 2k_l l_1$, $B_l = -2hl_1$, and $C_l = k_l^2 + h^2 + l_1^2 - l_2^2$, Eq. (7) can be written as

$$A_l\cos\theta_1 + B_l\sin\theta_1 + C_l = 0 \tag{8}$$

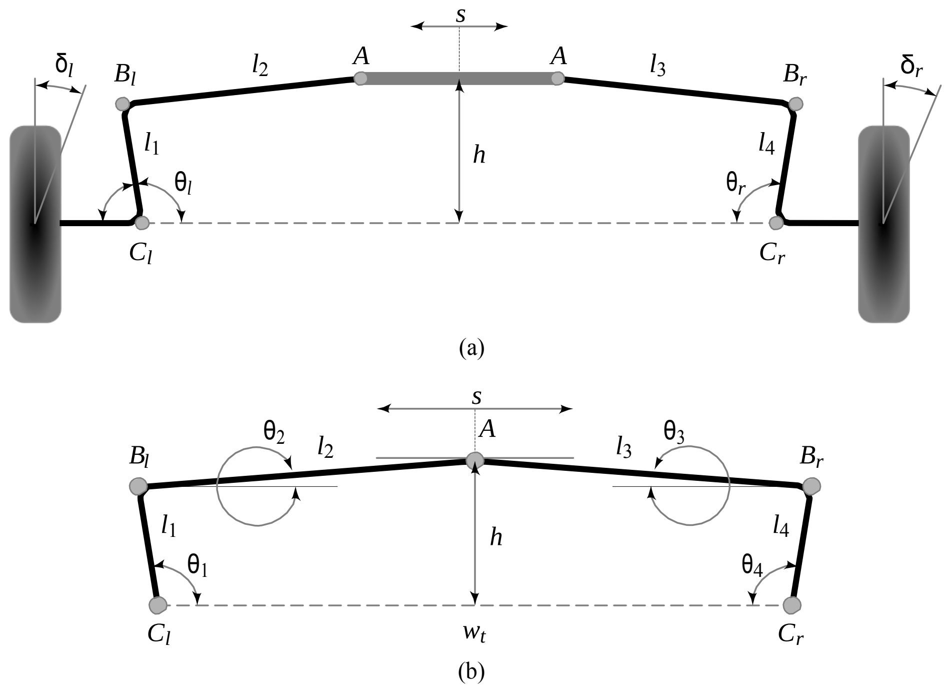


Fig. 3: Kinematic modeling of rack and pinion steering linkage

To solve Eq. 8 the trigonometric half-angle identities (Waldron and Kinzel, 2004) is used and then simplifying gives

$$A_l\left(1 - t_l^2\right) + B_l(2t_l) + C_l\left(1 + t_l^2\right) = 0 \tag{9}$$

where $t_l = \tan\left(\frac{\theta_1}{2}\right)$. Further simplification gives

$$(C_l - A_l)\,t_l^2 + 2B_l t_l + (A_l + C_l) = 0 \tag{10}$$

Solving for t_l gives

$$t_l = \frac{-B_l \pm \sqrt{B_l^2 - C_l^2 + A_l^2}}{C_l - A_l} \tag{11}$$

and

$$\theta_l = \theta_1 = 2\tan^{-1} t_l \tag{12}$$

Because $\tan^{-1} t_l$ has a valid range values $-\pi/2 \le \tan^{-1} t_l \le \pi/2$, θ_l will have the range $-\pi \le \theta_l \le \pi$. Typically, there are two solution for θ_l and they are both valid. These correspond to the two assembly modes. Therefore, we need to pick the corresponding desired mode. Using the same method, we obtain

$$t_r = \frac{-B_r \pm \sqrt{B_r^2 - C_r^2 + A_r^2}}{C_r - A_r} \tag{13}$$

and

$$\theta_r = \theta_4 = 2\tan^{-1} t_r \tag{14}$$

where $A_r = 2k_r l_4$, $B_r = -2hl_4$, and $C_r = k_r^2 + h^2 + l_4^2 - l_3^2$

Kinematic Analisys

The purpose of the current study is to analyze kinematically the rack and pinion steering linkage and to find the information on the steering linkage dimension and the placement of rack that give minimum steering error. The steering error is defined as (Simionescu and Smith, 2000)

$$\mathrm{SE} = |\delta_{oA} - \delta_o| \tag{15}$$

where δ_o is the actual angle made by the outer front wheel during steering maneuver and δ_{oA} is the correct angle for the same wheel based on the Ackermann principle given by Eq. 1. However, the angle θ_l and θ_r obtained from Eqs 12 and 14, are not the wheel angles. Define the condition for the wheels at the straight position as $\delta_l = \delta_r = 0$ and for the steering arms angle as θ_{l0}, θ_{r0}, then we have the following relation

$$\begin{aligned} \delta_l &= \theta_l - \theta_{l0} \\ \delta_r &= \theta_r - \theta_{r0} \end{aligned} \tag{16}$$

Table 1: Initial design parameters

Desciption	Notation	Values
Tie-rod length	L_t	36.5 cm
Steering arm length	L_a	12.6 cm
Distance of rack to wheel axis	h	14.7 cm
Wheel track	W_t	129 cm
Wheel base	W_b	259 cm
Rack length	L_r	64.5 cm
Steering arm angle	θ_l	109.18^0

First, let analyze the initial design of the rack and pinion steering linkage mechanism for the rural multi purpose vehicle. The linkages length for the initial design are shown in Table 1. Assume that the vehicle is turning to the right, we have $\delta_o = \delta_l$ and $\delta_i = \delta_r$. The relation between the left wheel angle δ_l, and the right wheel δ_r both for actual and Ackermann condition is shown in Fig. 4. It can be seen from this figure that steering error increases with the increases of wheel turning angle. Moreover, the maximum steering error is quite large around 4.5 degree, in this case.

Let now consider the effect of the rack placement h on the minimum turning radius and the steering error. In

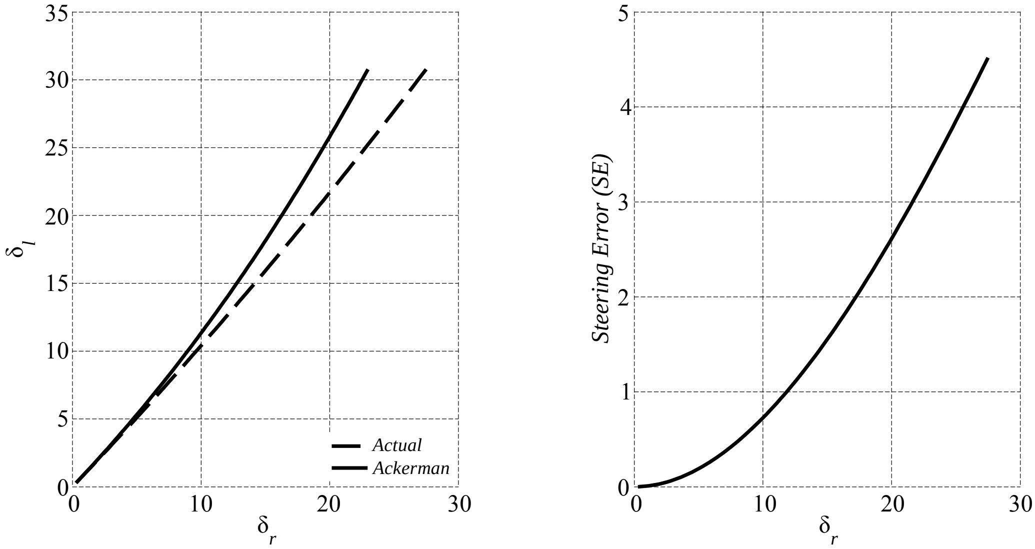


Fig. 4: Comparison between actual wheel angles with Ackermann wheel angles and the steering error

this case, the steering arm length is fixed. Therefore, to maintain the wheel angles unchanged for straight condition, we need to adjust the length of tie-rod. Based on Eq. 7 the length of tie-rod for different values of h is shown in Fig. 5. Fig. 6 shows the minimum turning radius for different values of h. It can be seen that by moving the rack closer to the front wheel axis, we obtain smaller turning radius. However, this condition is not always true for steering error, as shown in Fig. 7. We obtain minimum steering error for $6 < h < 8$ cm in front of the front wheel axis. Moreover, it is shown that the minimum steering error also depends on value of rack displacement s.

Next, let study the effect of steering arm length on the steering error. The same as the previous case, we use Eq. 7 to find the length of tie-rod when the steering arm length is varied. In this case the rack position h is fixed. Figs. 8 and 9 show the steering error for varied value of steering arm length. Fig. 8 is for the value of $h = 14.7$ cm which is the initial design, and Fig. 9 is for the value of $h = 7$ cm which represent the optimal rack placement from aforementioned analysis, respectively. It can be seen from Fig. 9 that the minimum steering error is obtained for the steering arm length = 12.6 cm, which is the same as the initial design. For the case $h = 14.7$ cm, the minimum steering error is greatly influenced by the rack displacement s.

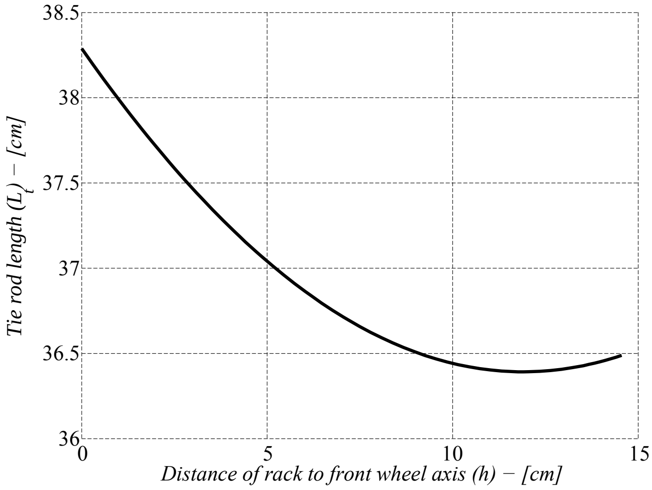


Fig. 5: Tie-rod length for different rack placement

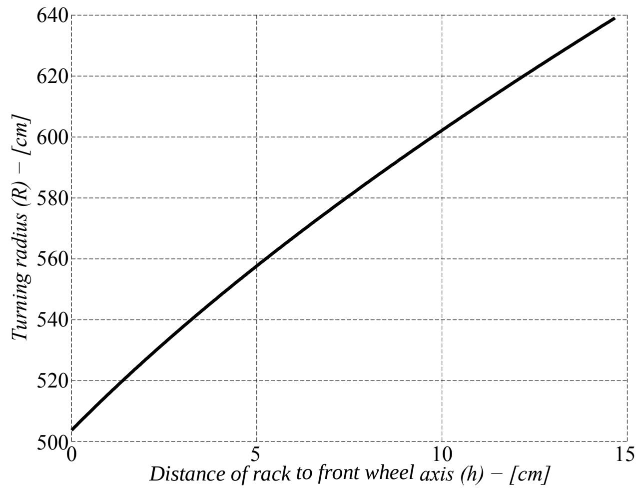


Fig. 6: Turning radius for different rack placement

Conclusion

From the analysis, it is found that placing the rack closer to wheels axis will reduce the turning radius of the vehicle. Moreover, by placing the rack around 6 - 8 cm in front of wheel axis will gives minimum steering error. Analysing the effect of steering arm length, it is found that for the value of $h = 7$ cm, the steering arm length that give minimum steering error is around 12.6 cm, which is the same as the initial design. The length of tie-rod depend on the placement of the rack and steering arm.

Acknowledgments

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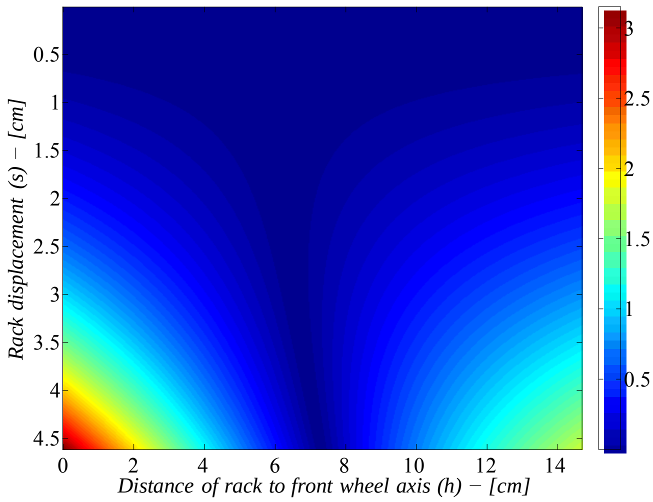

Fig. 7: Steering error for different rack placement and rack displacement

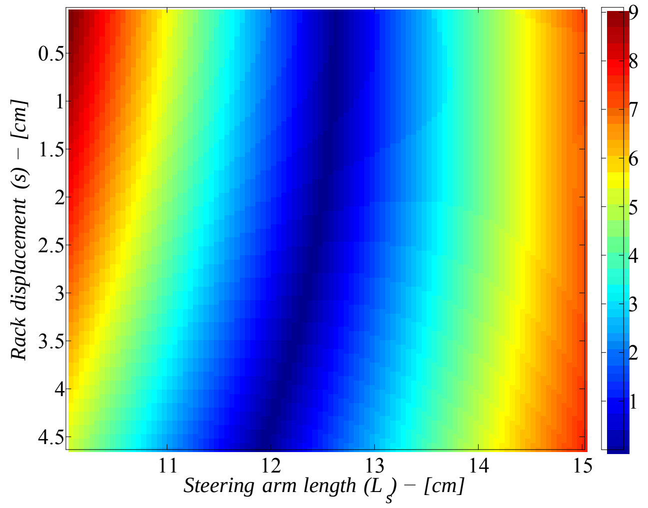

Fig. 8: Steering error for different steering arm length and rack displacement

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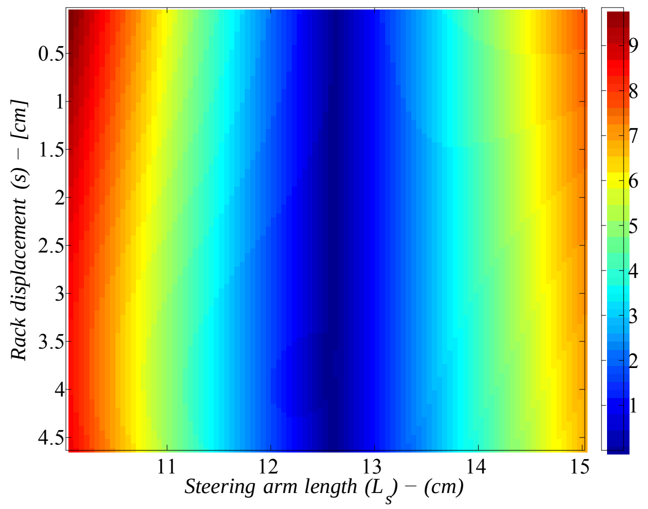

Fig. 9: Steering error for different steering arm length and rack displacement

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