M7-003 A novel tuning strategy for unconstrained model predictive control

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ABSTRACT

The most commonly used tuning strategy for a model predictive control (MPC) is by introducing a move suppression coefficient to prevent excessive changes in manipulated variables. However, this approach involves the iterative selection of certain parameters that are not well defined, and therefore demands sound understanding of the theoretical formulations. This procedure may lead to suitable process close loop responses. A more advanced approach is the use of constrained optimization methods which are computationally demanding in nature making it less suited for tight control of fast processes. In this paper, an effective tuning strategy for predictive control, i.e. shifting method, is proposed which leads to the reformulation of the original predictive control structure. The inherent ill-conditioning is eliminated by allowing the process prediction time step to be decoupled from the control time step. The original open loop data is used to evaluate a `shifting factor' m to be applied to the dynamic matrix structure, which replaces the move suppression coefficient.

The results show that the proposed tuning strategy gives improved closed loop responses for control simulations on a multivariable non-linear process having variable dead-time, and on other models found in the literature. The algorithm was also practically demonstrated on a fast reacting process and multi input multi output (MIMO) slow reacting plant, i.e. DC motor rotational speed control and a pilot scale distillation column, respectively, with better control being realized in comparison with DMC using move suppression. A major benefit of this proposed method is that only minor modification is required in order to implement this tuning strategy into the existing un-constrained control algorithm. It also eliminates the need for computationally intensive optimization of move suppression and uses purely open loop process data for tuning. The shifting factor m is generic, therefore it can be effectively applied for any control horizon and any processes.

Keywords: tuning strategy, dynamic matrix control, move suppression, shifting factor, shifted DMC

1. Introduction

Over the past decades, there have been tremendous advances in process control methodologies driven by the need for processes to become more efficient and cost effective. Further developments of modern control theory have resulted in its successful application in the aerospace and aircraft industries, as well as the manufacturing sector. The most significant advancement of modern control systems over the classical control theory is that they are not limited to single input single output (SISO) systems, but also available for a wide range of multi input multi output (MIMO) systems [1]. The significant improvement of digital computer performance, with prices falling considerably, has also contributed to the advances in process control.

The only advanced control methodology that has had a significant impact on industrial control engineering is predictive control [2].Predictive control is a model-based controller in the sense that it uses an explicit model obtained by applying linear system identification techniques to plant data, in order to predict the future plant behavior [3].

Over the years, several model based predictive controllers have been developed and implemented in a wide variety of applications including food processing, automotive, petrochemical refineries, aerospace and pulp and paper [4],[5],[6]. Among several algorithms belonging to the model based predictive controllers, dynamic matrix controller (DMC) is arguably the most popular advanced control strategy employed in industry to date [7],[8],[9],[10]. It has been implemented and tested with much success on a wide variety of applications over the last decades. The DMC tuning strategy involves a number of adjustable parameters that can affect the process closed-loop dynamics. These include the prediction horizon P, control horizon n_u, model horizon N, sampling interval T and move suppression coefficient λ.

The tuning strategies range from systematic trial and error [11] to formal tuning strategies such as move suppression coefficient [12] and input blocking [13]. A method of selecting the prediction horizon P, model horizon N and control horizon n_u was presented by Cutler [14]. It was proposed to set $P=N+ n_u$ and then continually increase n_u until further changes in n_u have no further effect on the first control move in the evaluated control vector. The main tuning variable is then a move suppression coefficient λ which is then chosen iteratively. Increasing λ reduces the size of the manipulated input changes generated by the controller and hence, slows down the closed-loop response.

An off-line unconstrained DMC tuning strategy for SISO and MIMO processes was proposed by Shridhar and Cooper [15][16]. They confirmed that the move suppression λ can serve as the primary adjustable parameter in DMC tuning for fast and slow reacting processes. An analytical expression to compute λ was derived irrespective of the choice of other tuning parameters resulting in modest manipulated variable changes, and therefore relatively slow close loop transients.

A simple and an effective adaptive tuning of move suppression λ was developed for an injection molding sub cycle, i.e., screw speed rotation and melt temperature control [17]. The process response is close to critical damping during its transient state, therefore providing fast settling and reduced overshoot. The only drawback with this method is the evaluation of the first value of λ which depends on initially calculating the closed-loop variance after three or more samples of the plant output before active implementation.

Move suppression λ was also demonstrated to be an effective tuning parameter for integrating (non-self regulating) processes [10]. The controller exhibited a good setpoint tracking without overshoot, fast rise time as well as effective disturbance rejection.

However, the application of this tuning strategy was demonstrated only through simulation.

A wealth of information relating the changes in these tuning parameters and their effects on the closed-loop response, indicate a continued interest towards tuning strategies of model predictive controllers. It is clear that in order for equipment and process operators to implement the more advanced tuning strategies, sound knowledge of the advanced predictive control concepts and the corresponding tuning strategy must be acquired. This is particularly due to the presence of ambiguous variables used in these advanced tuning schemes, such as the move suppression λ, and therefore demands very good understanding of the theoretical formulations for their iterative selection. Moreover, most of the tuning methods are computationally demanding and hence are less suited to control of fast processes. Therefore, it is justified to say that there is a need for a simple and effective tuning strategy which is not computationally demanding, requiring minimum effort for implementation.

2. Predictive Control Theory

Predictive control which also known as model-based predictive controllers (MPC) can be illustrated in Fig. 1. The future outputs are predicted at each sampling instant *t* using a process model. The prediction outputs have two main components, the free response and the forced response. The free response is the expected behavior of the plant due to past inputs and outputs assuming there is no future control actions or deviations. The forced response is the additional component of the output response due to a set of future control actions.

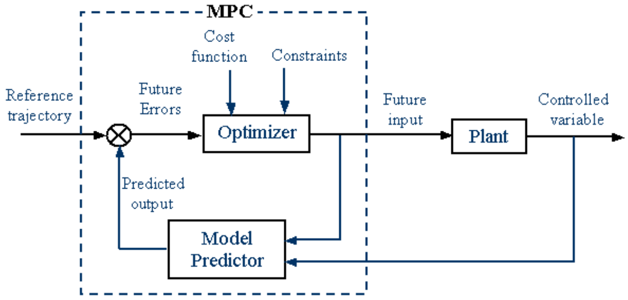


Figure 1. General MPC architecture

The difference between the predicted outputs and the reference trajectory is termed future errors which are to be minimized, taking into account the cost function as well as the constraints. The future errors can be illustrated more detail in Fig 2. These errors are formulated in an objection function subjected to given constraints in the magnitude of the control moves and its changes. The least squares method is used for the error minimization since the number of unknowns exceeds the degrees of freedom.

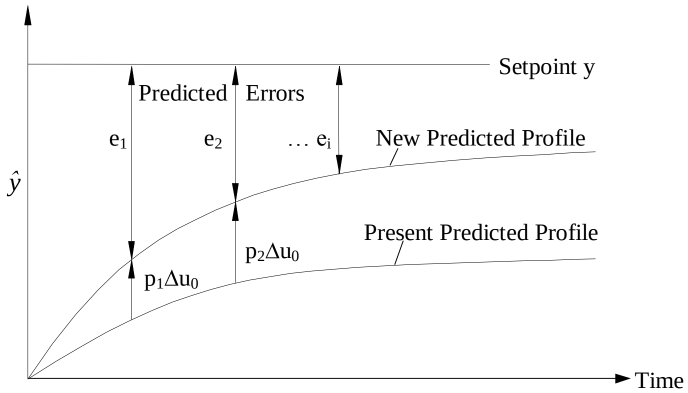


Figure 2. Predicted errors over the prediction horizon *P*

The objective function can be expressed as follows,

$$J_{MPC} = \min\left\{ \sum_{k=1}^{P}\left[y(t+k) - \hat{y}(t+k\,|\,t)\right]^2 + \lambda \sum_{k=1}^{n_u}\left[\Delta u(t+k-1)\right]^2 \right\} \quad (1)$$

The first term in Eq. 1 is the difference between the desired trajectory vector y and the predicted process output $\hat{y}$ giving the future vector of errors *e*, over a prediction horizon *P*. This horizon represents the number of discrete sampling instants that the process is predicted in the future. The second term is the changes in the control moves or manipulated variables, which are to be evaluated over a control horizon n_u. The value of *P* is based on an open loop testing using a step or multi-step inputs to the process to be controlled. The manipulated variable changes in Δu are evaluated over a control horizon n_u which is the number of manipulated variables changes to be determined in the future. The evaluation of Δu depends upon the process response obtained from open loop tests. This response is then normalized and expressed as a dynamic matrix *A* in Eq. 2 for SISO control.

$$A = \begin{bmatrix} a_1 & 0 & \cdots & 0 \\ a_2 & \cdots & \cdots & 0 \\ \vdots & \cdots & \cdots & \vdots \\ a_P & a_{P-1} & \cdots & a_{P-n_u+1} \end{bmatrix}_{P*n_u} \quad (2)$$

The move suppression coefficient λ is introduced to avoid excessive changes in the manipulated variable due to the ill-conditionality of the system matrix $A^T A$. Using the least squares method, Eq. (2) yields a solution known as the DMC control law,

$$\Delta u = \ A^T A + I\lambda\ ^{-1}\ y - \hat{y} \quad (3)$$

subject to the following constraints,

$$\begin{aligned} \Delta u_{\min} \le \Delta u \le \Delta u_{\max} \\ u_{\min} \le u \le u_{\max} \end{aligned} \quad (4)$$

Using Δu the prediction of the process output or controlled variable is calculated using

$$\hat{y}(t+k\mid t)=\hat{y}(t)+\sum_{i=k-n_u+1}^{k} a_i \Delta u(t+k-i)+\sum_{i=k+1}^{k+P-1}(a_i-a_{i-k})\Delta u(t+k-i)+\phi(t) \quad (5)$$

with $\phi(t)$ an adjustment parameter that accounts for process non-linearities. Further details of the DMC control theory can be found in [14].

3. Proposed Tuning Strategy

The DMC algorithm uses normalized open-loop response coefficients to form the dynamic matrix A for its process predictions as well as future errors calculations. Open loop response data is obtained by inputting defined changes of the manipulated variable, and then sampling the process as fast as possible with respect to the practical limitations of the system. This approach is generally used in order to capture the full dynamics of the process, and therefore provides an accurate representation of the continuous response. These sampled and normalized response coefficients form a dynamic matrix A, in which the difference ε between elements of the corresponding rows in A or corresponding columns in A^T is generally very small. In predictive control such as DMC, the evaluation of Δu without move suppression depends on A^TA. Since ε is small, the matrix A^TA becomes highly ill-conditioned. In other words, closed-loop control becomes increasingly ill-conditioned as the sample time, Δt, is reduced, resulting in the manipulated variables becoming excessively large. It is now clear that the ill-conditioning of A^TA can be reduced by increasing ε.

Define a shifting factor m>1 to be applied to the dynamic matrix A where the second column of A is shifted downwards by m>1 followed by the same for subsequent columns. Now the re-formulation of A of a SISO process in terms of the shift m, for n_u=2, gives Eq.6

$$A=\begin{bmatrix} a_1 & 0 \\ \vdots & \vdots \\ a_m & 0 \\ a_{m+1} & a_1 \\ \vdots & \vdots \\ a_{m+P} & a_P \end{bmatrix} \quad (6)$$

Where *m-1* extra zeroes in the second column are indicative of the *m*-shifting. A^TA immediately follows as

$$A^TA=\begin{pmatrix} \sum_{k=1}^{m+P} a_k^2 & \sum_{k=1}^{P} a_k a_{m+k} \\ \sum_{k=1}^{P} a_k a_{m+k} & \sum_{k=1}^{P} a_k^2 \end{pmatrix} \quad (7)$$

If Δt is the sampling time and let $T\equiv P\,\Delta t$, $\omega\equiv m\,\Delta t$ and consider $\Delta t\rightarrow 0$, then for a prediction horizon $P>>m$, A^TA can be expressed as

$$A^TA = \frac{1}{\Delta t}\begin{pmatrix} \int_0^{T+\omega} a^2(t)dt & \int_0^{T} a(t)a(t+\omega)dt \\ \int_0^{T} a(t)a(t+\omega)dt & \int_0^{T} a^2(t)dt \end{pmatrix} \quad (8)$$

Using linear algebra manipulation, Eq. 8 yields Eq. 9 for large *T*

$$\left(A^TA\right)^{-1} \approx \frac{2\Delta t}{\omega^2}\left(1-\frac{3}{2T}\right)\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \quad (9)$$

By shifting the 2nd downward by *m*, the second prediction during predicting the future errors will then be shifted to right by *m* time units, as illustrated in Fig. 3.

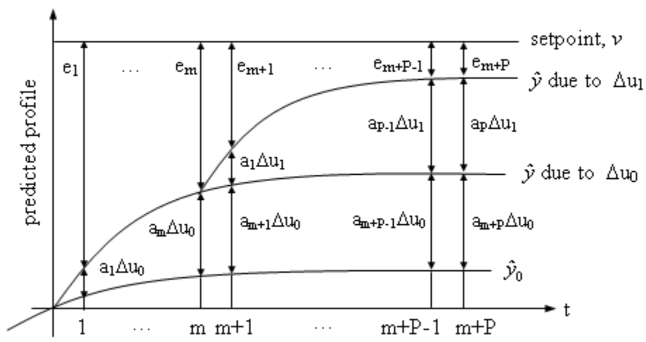


Figure 3. Predicted errors over the prediction horizon *P* after *m* shift

Using the same approach as the original DMC, the solution of the errors minimization is given by

$$\Delta u = (A^TA)^{-1}A^T(y-\hat{y}) \quad (10)$$

It is clear that calculated manipuated variable obtained from Eq. 10 yield the original DMC control law without move suppression. Therefore, implementation of the new approach to the existing DMC algorithm can be easily performed.

4. Results and Discussions

The validation of the proposed algorithm was demonstrated through control simulations of several higher order SISO processes from [9] and MIMO pilot scale distillation column. Practical implementation was also conducted on a DC motor speed control system. The plants used for control simulations were

Process 1: $G_p(s) = \frac{(60s+1)e^{-10s}}{(100s+1)^2}$ (11)

Process 2: $G_p(s) = \frac{e^{-10s}}{(100s+1)^4}$ (12)

Process 1 represents a plant with minimum phase behaviour while process 2 is a fourth order process with sluggish open loop dynamics. First order plus deadtime (FOPDT) approximation to the above processes yield the following process parameters:

- $Kp = 1$, $\tau_p = 148$, $\theta_p = 18$ *and* $\lambda = 0.14$, $m = 2$ (process 1)
- $Kp = 1$, $\tau_p = 124$, $\theta_p = 99$ *and* $\lambda = 0.15$, $m = 3$ (process 2)

Figures 4,5 and 6,7 show the results of the algorithms for performing control simulations for both shifted and move-suppressed DMC application and its corresponding manipulated variables for processes 3 and 4, respectively. It can be seen that introducing the shifting factor *m* is simple and effective while yielding stable process responses with no excessive changes in the control moves, yet faster than the original DMC using move suppression factor.

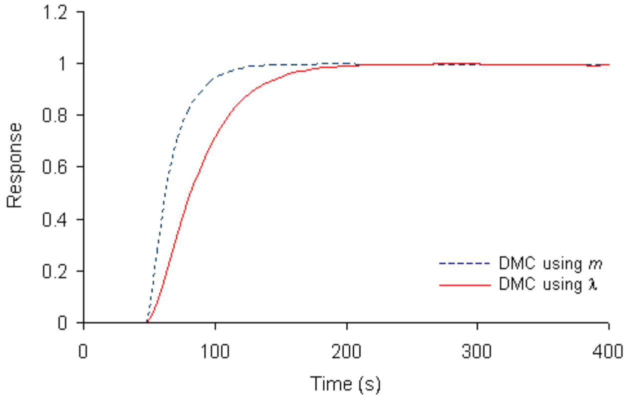


Figure 4. Process 1 response using shifted and move-supressed DMC

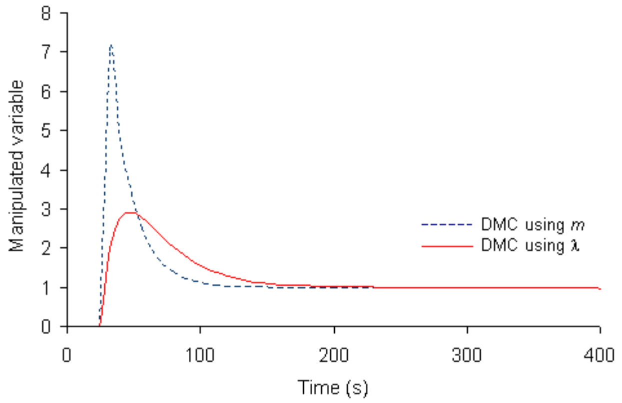


Figure 5. Process 1 manipulated variable using shifted and move-supressed DMC

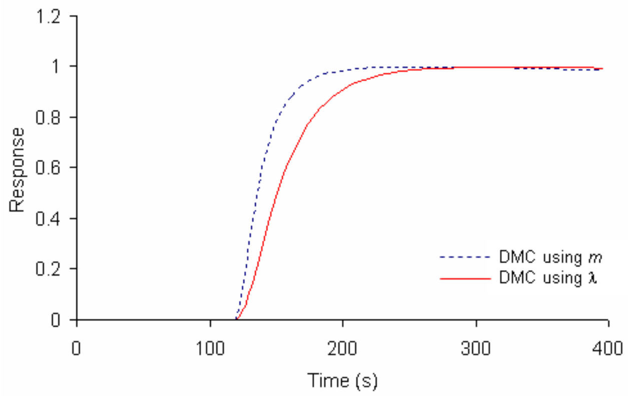


Figure 6. Process 2 response using shifted and move-supressed DMC

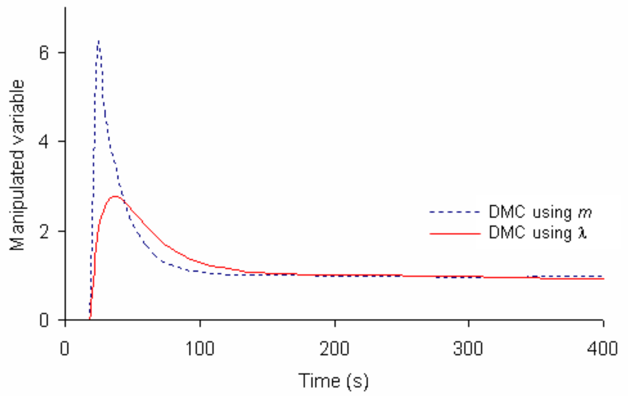


Figure 7. Process 2 manipulated variable using shifted and move-supressed DMC

The proposed algorithm was also implemented on a much harder to control process, i.e. MIMO distillation column, developed by Wood and Berry [18] which has been extensively used in the literature. The distillation column plant is given by,

$$\begin{bmatrix} y_1(s) \\ y_2(s) \end{bmatrix} = \begin{bmatrix} \frac{12.8e^{-s}}{16.7s+1} & \frac{-18.9e^{-3s}}{21s+1} \\ \frac{6.6e^{-7s}}{10.9s+1} & \frac{-19.4e^{-3s}}{14.4s+1} \end{bmatrix} \begin{bmatrix} u_1(s) \\ u_2(s) \end{bmatrix} \quad (13)$$

Figure 8 shows the application of shifted DMC on the plant given by Eq. 13 with good results. A shifting factor *m* of 20 was used for control simulation of distillation colums.

(9)

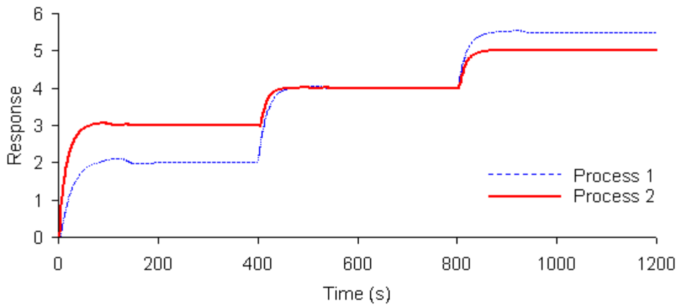
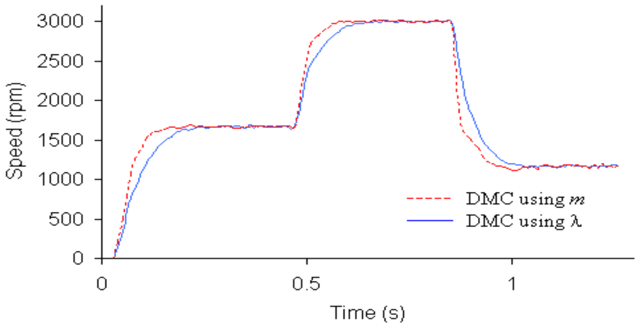


Figure 8. Response of distillation column plant

For real time application, a defined input was sent to the motor DC speed control to obtain open loop response coefficients to form the dynamic matrix *A*. Closed-loop testing was then conducted using the move-suppressed DMC with λ=0.07, as well as with the shifted DMC with *m*=12. The process responses in Fig. 9 indicate much shorter rise time and settling time with no oscillations when using the proposed method. It is worth mentioning that the relative ease of tuning becomes apparent since λ can vary by as little as of 0.001, as compared to *m* varying by 1.



. Figure 9. Speed control of DC motor using shifted and move-supressed DMC

5. Conclusions

A new approach of tuning predictive control using a new variable termed shifting factor *m* is developed. This new method reformulates the fundamental design of original unconstrained dynamic matrix control by de-coupling the prediction and control time steps. The application of the proposed method on both control simulation and real time demonstrate better control performance as compared to the move-suppressed DMC. Reduced rise and settling time as well as minimum overshoot were achieved when implemented for either SISO or MIMO plants. This method is also readily introduced to the existing algorithm with minor modification.

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