

Numerical Study on Onset of Convection in a Porous Medium

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Abstract

The onset of convection in a porous medium has been studied numerically. A three-dimensional hypothetical geothermal system is utilized as the model. The system is divided into 100 grid blocks and each block has a dimension of 100m x 100m x 100m. The heat source subjected at each block of the system is varied in the range of 1000 – 8000 W to find the minimum subjected heat that will generate onset of convection. The calculated results show that for the considered system, the onset of convection is reached when the system is subjected to the heat source of 2000 W at the base which corresponds to the temperature of 100°C.

Keywords: onset of convection, porous medium, heat source, grid block

Introduction

A number of researches have been carried out on porous media. The application areas of this field can be insulation for buildings and equipments, energy storage and recovery, nuclear waste disposal, chemical reactor and geothermal reservoir. This paper will focus on geothermal application.

In geothermal application, the numerical study of convection in porous media is as part of geothermal reservoir modeling. The geothermal reservoir modeling, basically, can be divided into; reservoir modeling at natural state condition and at production stage. From the natural state modeling we will understand about the estimation of power potential, pressure and temperature distribution in the reservoir, the properties of porous medium, etc. While from the production stage modeling, it can be found the information on the pressure and temperature changes, the scaling problems, mass flow rate change, etc. This paper discusses the natural state modeling.

The aim of this study is mainly to investigate the onset of convection in a given porous medium with known dimension due to various heat sources subjected at the base of the system. The problem of onset of convection is considerably important because it is one of the problems involving hydrodynamic instability. The onset of convection is the transition between conductive and convective conditions. The situation where the motionless state of the saturating fluid exists is the conductive condition. It is characterized by the uniform temperature in the entire system. On the other hand, when the temperature of the saturating fluid is not uniform, some flows induced by buoyancy effects will occur. The onset of convection is required to study because it is one of the important phenomena in the geothermal application. It will help us to decide which locations that must be drilled for production wells.

Development of Model

The model of the considered system is shown in figure 1. The system consists of a porous medium filled with fluid and involves free thermal convection in a block of reservoir. The system has a dimension of 1000m x 1000m x 100m and is divided into 100 grid blocks. The upper boundary of the system is maintained at a constant pressure of P_o and a constant temperature of T_o . The bottom boundary is subjected to a constant heat flux at each block. Both sidewalls of the system are thermally and hydrodynamically insulated. The problem is solved by simulator, namely MULKOM.

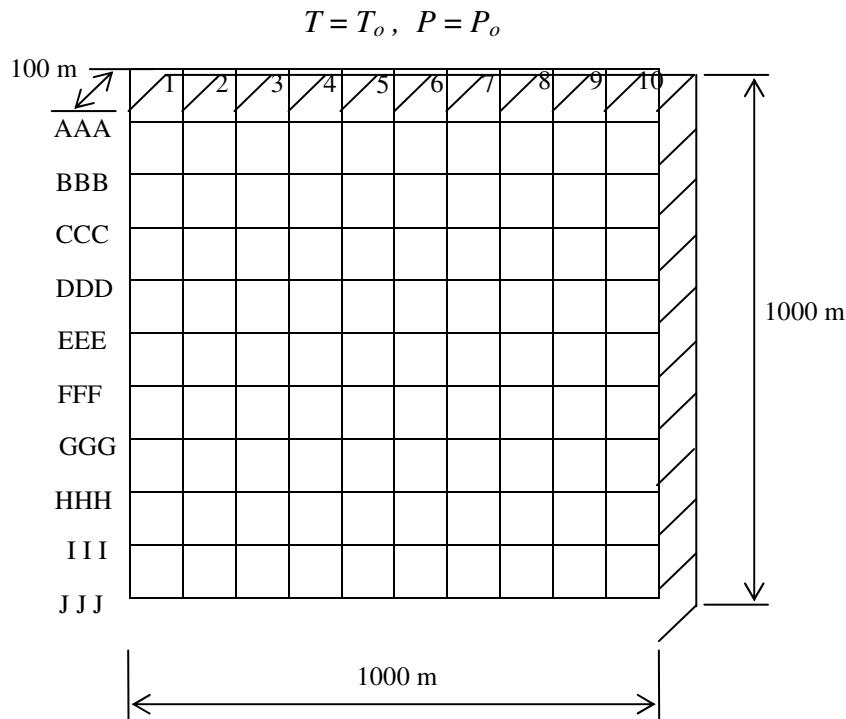


Figure 1. Schematic diagram of the model.

Governing Equations

The governing equations solve coupled mass and energy balances. The assumptions for solving the equations are as follows;

- The physical system is approximated as system as porous rock saturated with one-component fluid in liquid and vapor form.
- Except for porosity which can vary with pressure and temperature all other rock properties; density, specific heat, thermal conductivity, absolute permeability are independent of temperature, pressure or vapor saturation.
- Liquid, vapor and rock matrix are in local thermodynamic equilibrium, i.e., at the same temperature and pressure at all times.
- Capillary pressure is neglected.

The equations consist of the following expressions;

1. Mass Balance Equation

$$\frac{\partial \varphi \rho}{\partial t} = -\operatorname{div} F + q \quad (1)$$

where, φ = porosity (void fraction)
 ρ = fluid density
 F = mass flux
 q = a source term for mass generation

2. Energy Balance Equation

$$\frac{\partial u}{\partial t} = -\operatorname{div}G + Q \quad (2)$$

where, u = volumetric internal energy of the rock/fluid mixture
 G = energy flux
 Q = an energy source term

Mass flux is given by Darcy's Law:

$$E = \sum E_\alpha = -\sum_\alpha \frac{KK_\alpha}{\mu_\alpha} \rho_\alpha (\Delta p - \rho_\alpha g) \quad (3)$$

where, K = absolute permeability
 K_α = relative permeability
 μ_α = viscosity
 p = pressure
 g = gravitational acceleration

Energy flux contains conductive and convective terms:

$$G = -k\Delta T + \sum_\alpha h_\alpha F_\alpha \quad (4)$$

where, k = thermal conductivity of the rock/fluid mixture
 T = temperature
 h_α = specific enthalpy of vapor ($\alpha=v$) or liquid ($\alpha=l$)

Results and Discussion

The minimum value of heat source Q for which the system is in the first convective condition is evaluated by using the typical values for physical properties of medium (rock) and fluid (water) as follows,

$$\begin{aligned} T_o &= 15^\circ\text{C} & \varphi &= 0.1 \\ \rho &= 1000 \text{ kg/m}^3 & K &= 10^{-14} \text{ m}^2 \\ k &= 2.5 \text{ W/m}^\circ\text{C} & v &= 10^{-7} \text{ m}^2/\text{s} \\ g &= 9.8 \text{ m/s}^2 \end{aligned}$$

The numerical results for the relationship between heat generation against temperature at several grid points are presented in figure 2. It can be seen that when heat source is increased from 1000W to 2000W in each block at the base, the temperatures at all grid points in the system are the same. In other words, the temperature distribution at the same layer (depth) is uniform. It can be concluded that the mode of heat transfer when the system is subjected to heat source up to 2000W for each block at the base is conduction. The bottom temperature at this condition is about 100°C.

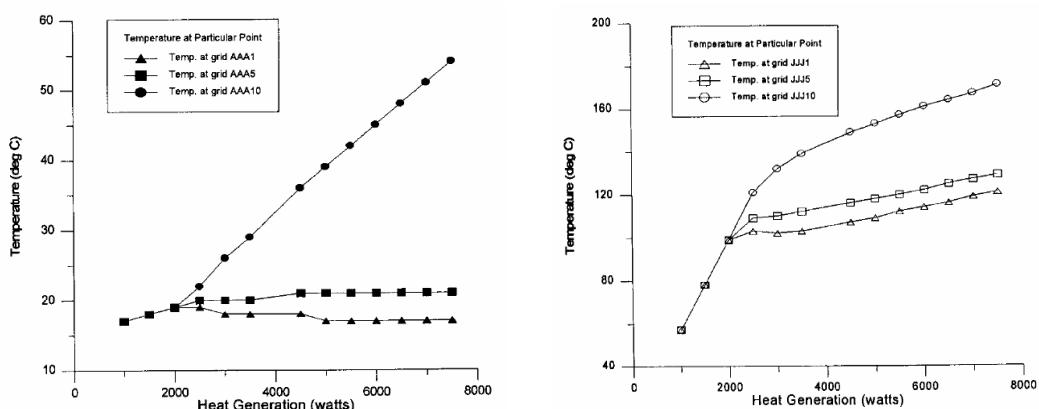


Figure 2. Temperature distribution at different grid points for various heat generations.

When the heat source is increased from 2500W for each block at the base, the temperature at the same layer is not uniform anymore. Thus, it can be concluded that the mode of heat transfer is convection. For the given system, the critical condition (onset of convection) lies between 20000W and 25000W of total heat source at the base is observed. This corresponds to the temperatures at the base in between 100°C and 110°C.

In geothermal application, beside the heat source as a key parameter, another important consideration is how much fluid can be produced from the system. The mass flow rates produced for respective heat source at particular points are shown in figure 3.

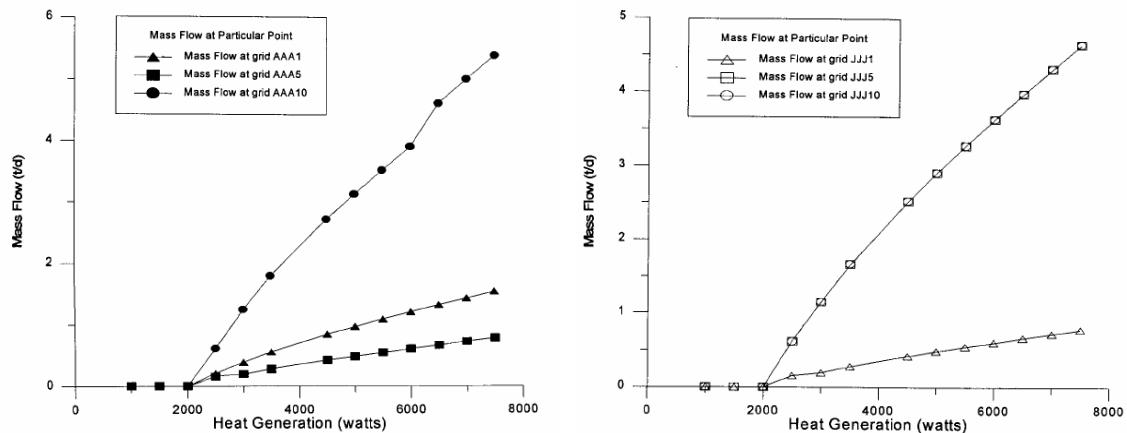


Figure 3. Mass flow rate distribution at different grid points for various heat generations.

It can be seen that during the conductive process, there is no fluid motion is observed. The minimum flow at the surface layer is at grid AAA5 which is in the middle of cell width is observed, while the maximum flow is at somewhere in column 10 (below hot spot).

Conclusions

1. The numerical simulation is more applicable in evaluating the convective system due to the ability to solve a complex system.
2. The numerical study on onset of convection for a given system consisting of porous medium filled with fluid may be used to estimate the heat source subjected to the base of system.

References

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