Stress Concentration Factors of Stepped-Shafts of Circular-to-Square Cross Section under Twisting Moment

Asnawi Lubis[1] & Jamiatul Akmal
Jurusan Teknik Mesin, Universitas Lampung
Gedung H Fakultas Teknik, Jalan Professor Sumantri Brojonegoro
Gedongmeneng, Bandar Lampung, 35145
Telp: (62-721) 7479221, 701609 ext. 221, Fax: (62-721) 704947
[1]E-mail: asnawi-lubis@unila.ac.id

Abstract

Shaft is a machine element whose main function for power transmission. Shaft is often needed in stepped form (change in cross-section). For a non-smooth shaft (stepped shaft), the elementary stress equations are no longer hold for calculating the actual stress as the stress would be higher than the nominal stresses. To relate the maximum stress at the discontinuity with the nominal stress calculated using the elementary stress equation, a factor is needed which is called stress concentration factor (SCF). This paper presents a graph for design purpose of stress concentration factor of stepped shaft of circular-to-square cross-section under twisting moment. The graph was derived from an extensive linier finite element parametric study and presented in line of the well-known Peterson's graph for stepped shaft of uniform circular cross-section. Finite element parametric study was carried out for every combination of diameter to diagonal ratio, D/d and radius of fillet to diagonal ratio, r/d. For geometry with particular D/d value, six analyses were done by varying the r/d value. Stress distribution shows that maximum shearing stress located at the root of square, exactly at midside. In general graphical design similar to the well known Peterson graph was produced.

Keywords: stepped-shafts, stress-concentration factors, finite element analysis

Introduction

Shaft is a machine element whose function for power transmission. Shaft is often needed in stepped form (change in cross-section). Example is a rotated shaft with square shoulder for the seat of bearing. In such, the shaft could be subjected to combined loading of axial, bending, and torsion.

If a shaft is smooth, the resulting stress due to loading can be computed using the elementary stress equations. This stress is called nominal stress. For a non-smooth shaft (stepped shaft), the elementary stress equations are no longer hold for calculating the actual stress as the stress would be higher than the nominal stresses. To relate the maximum stress at the discontinuity with the nominal stress calculated using the elementary stress equation, a factor is needed which is called stress concentration factor (SCF). The magnitude of this factor depends on diameter ratio, shape and area of cross-section, radius of fillet and types of loading (axial, bending, torsion, or their combination).

Stress concentration factors for a stepped shaft of uniform cross-section can be found in many standard textbooks of mechanics of materials. Until now, the most complete source of graph of stress concentration factors for various geometry of shaft, bar, and plat is a compendia by Peterson [1974]. In this compendia, stress concentration factor (K_t) for stepped shaft of uniform cross-section are plotted as function of non-dimensional geometry. Today state of the art modern literature are also still using the Peterson's graph for analyzing the stepped shaft of uniform cross-section, see for example, Kang, et al [2005]. Pan, et al [2000] used the Peterson's graph to analyze the torsional vibration of stepped shaft. Kim and Kim [1995] used the same graph to analyze the stresses at the stepped shaft subjected to axial tension.

In the compendia of stress concentration factors by Peterson, a graph for stepped shaft of circular-to-square cross-section is not available. Baker [1999] carried out a finite element study using

ALGOR version 3.18 to obtain the characteristic of stepped shaft of circular to square cross-section subjected to combined loading of static bending and torsion. In his MSc thesis, Baker investigated three values of diameter ratio, but did not make the radius of filled varied so graphs that can be used for design purpose were not produced. To analyze a stepped shaft of circular-to-square cross-section, it is common to apply the graph for stepped shaft of uniform circular cross-section of equivalent cross-sectional area to square [Yusron, 2004]. This was usually done because similar graph for stepped shaft of circular-to-square cross-section was not available either in textbooks or in today state of the art modern literature. However, current study on stress concentration factor of stepped shaft of circular-to-square cross-section under axial tension and pure bending was presented in two papers by Lubis and Akmal [2006]. The present paper is a continuation of that study where stepped shaft of circular-to-square cross-section was subjected to static twisting moment and a graph for stress-concentration factor was produced. The graph was derived from a series of extensive linier finite element parametric study and presented in line of the well-known Peterson's graph for stepped shaft of uniform circular cross-section.

Stress-Concentration Factors

Shaft is an important element in mechanical construction. Shaft is usually has function for power transmission. Shaft can be found as square cross-section as well as circular cross section. However, majority of shaft are circular cross-section as its main function is for power transmission, whereas, shaft of square cross-section is usually only used for supporting load.

Circular shaft is a rotating element where other elements such as gears and pulley are mounted. Such a shaft could be subjected to axial (tension or compression), bending, torsion, or their combination as shown in Figure 1.
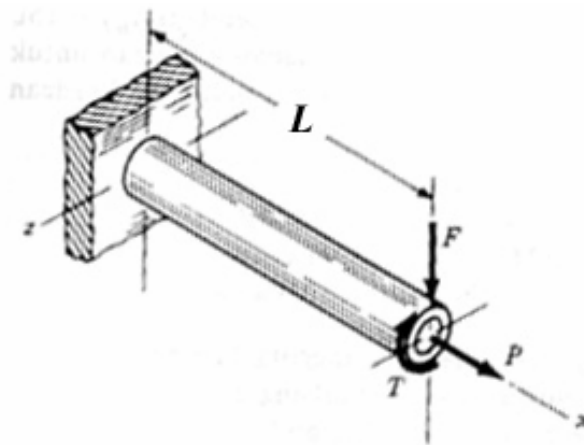

Figure 1 A circular shaft subjected to combined loading of axial, bending and torsion

In particular application, a shaft of non-uniform area of cross section (stepped shaft) might be used. Change in area of cross-section of stepped shaft results in stress concentration at the discontinuity. In order to reduce the effect of stress concentration at the region of change in cross-section, fillet of radius r is usually applied as shown in Figure 2.
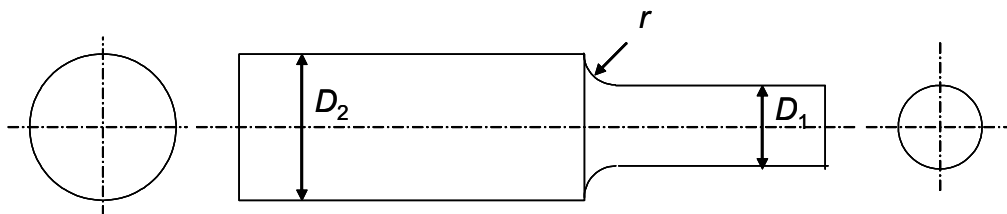

Figure 2 Circular stepped shaft with shoulder or radius r

The ratio of maximum stress at a critical point to the nominal stress is called stress concentration factor (*SCF*). Stress concentration factor of circular stepped shaft has been given by Peterson [1974]. The magnitude of this factor is a function of non-dimensional parameter D_2/D_1 and r/D_1. Stress concentration factor, K_t, can be written as:

$$K_t = \frac{\sigma_{max}}{\sigma_{nom}} \text{; For normal stress} \tag{1a}$$

$$K_{ts} = \frac{\tau_{max}}{\tau_{nom}} \text{; For shear stress} \tag{1b}$$

Where, for circular cross section the nominal stress is given by:

$$\sigma_{nom} = \frac{4P}{\pi D_1^2} \text{; For axial load} \tag{2}$$

$$\sigma_{nom} = \frac{32M}{\pi D_1^3} \text{; For bending load} \tag{3}$$

$$\tau_{nom} = \frac{16T}{\pi D_1^3} \text{; For torsion load} \tag{4}$$

Figure 3 shows a typical graph of stress concentration factor for circular stepped-shaft under twisting moment. The graph was taken from a book of "Stress Concentration Factor" by Peterson [1974]. Similar graph for axial and bending load can be obtained from the same book.
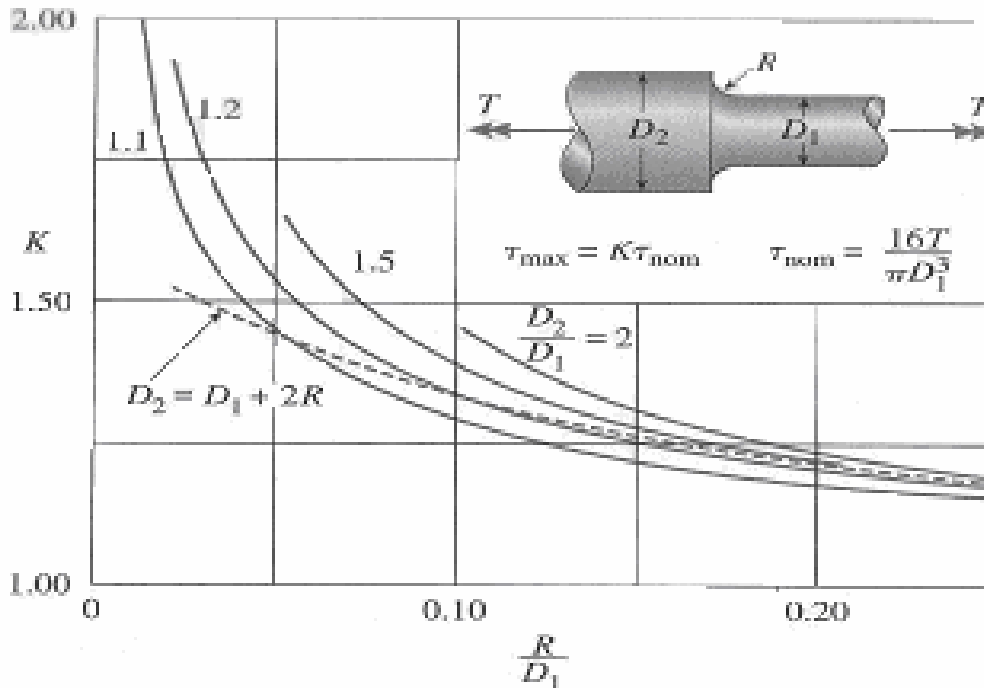

Figure 3 Stress-concentration factors for circular stepped-shaft under torsion [Peterson, 1974]

In the book of Peterson, it cannot be found a graph of stress-concentration factor for a stepped shaft of circular-to-square cross-section (Figure 4). In fact, stepped shaft of this form can be found in practice, such as, the output low speed reducer at mill and diffuser station in sugar cane industry [Yusron, 2004]. This paper aims to provide a graph of stress concentration factors of stepped shafts of circular to square cross-section under twisting moment that can be used directly for design purpose.
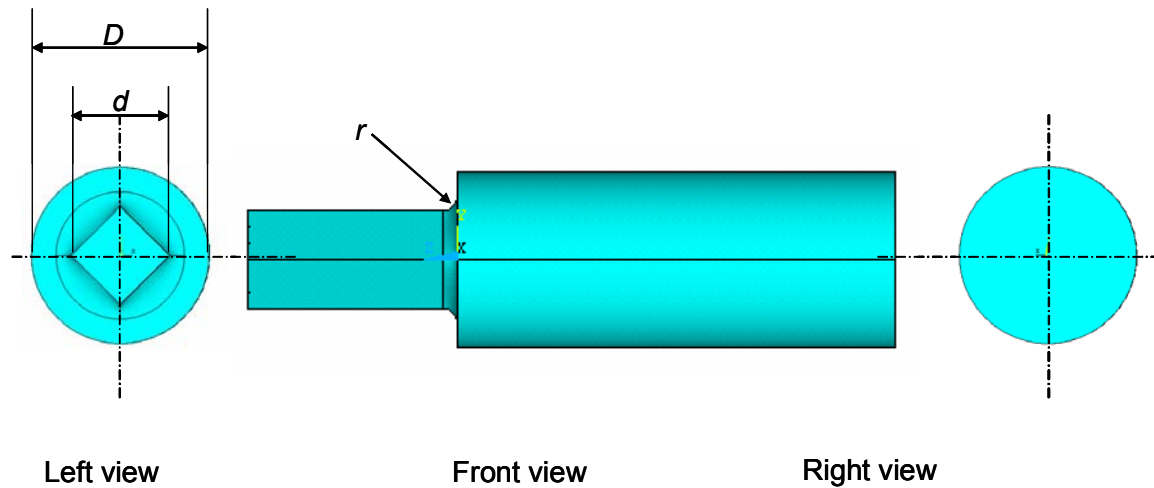

| Left view | Front view | Right view |

Figure 4 Stepped shaft of circular to square cross-section

Finite Element Analysis

Figure 4 shows a stepped shaft of circular cross section of diameter *D* to square cross-section of diagonal *d*. Finite element parametric study was carried out for every combination of parameter *D/d* and *r/d*. For geometry with particular *D/d* value, six analyses were done by varying the *r/d* value. Table I gives the geometry configuration for parametric finite element study performed.

Table I Geometric configuration for parametric study (see Figure 2)

D/d	*r/d*					
1.1	0.05	0.10	0.15	0.20	0.25	0.30
1.2	0.05	0.10	0.15	0.20	0.25	0.30
1.5	0.05	0.10	0.15	0.20	0.25	0.30
2.0	0.05	0.10	0.15	0.20	0.25	0.30
2.5	0.05	0.10	0.15	0.20	0.25	0.30
3.0	0.05	0.10	0.15	0.20	0.25	0.30

Figure 5 shows a typical finite element model of the shaft. The element type used has ten nodes with three degree of freedom at each node: translation in the -x, -y, and –z direction. All nodes at the circular end were fixed and twisting moment was applied at the central node of the square cross section end. The central node was connected to all nodes at the four sides of the square using beam element of relatively higher stiffness than the element type for the shaft.
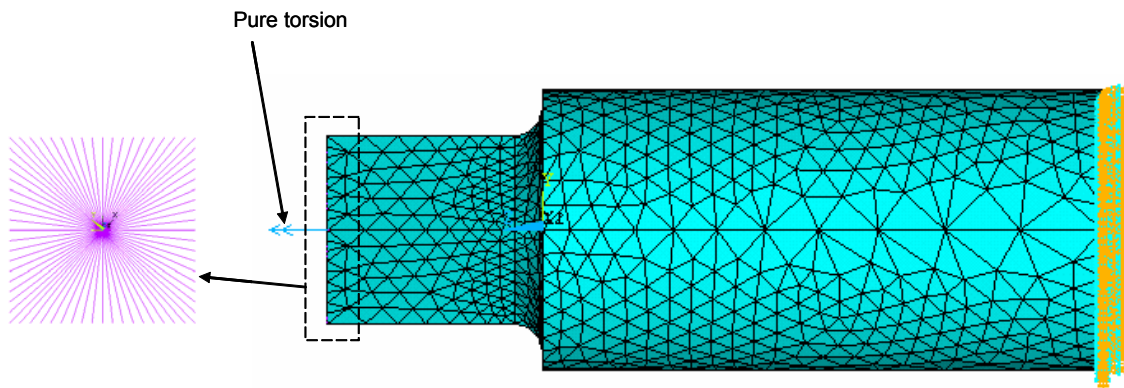
Figure 5 Typical finite element model of stepped shaft of circular to square cross-section

Figure 6 shows the shear stress distribution along the shaft due to pure torsion. The maximum shear stress can be read from the graph. This maximum stress was then recorded for every combination of parameter *D/d* and *r/d* and divided by the nominal stress for twisting moment [see equation (5)]. From this, a graph in line of Peterson's for uniform circular stepped shaft can then be plotted.

$$\tau_{nom} = \frac{T}{0.208s^3} \qquad (5)$$

Where, *T* = twisting moment, N.mm
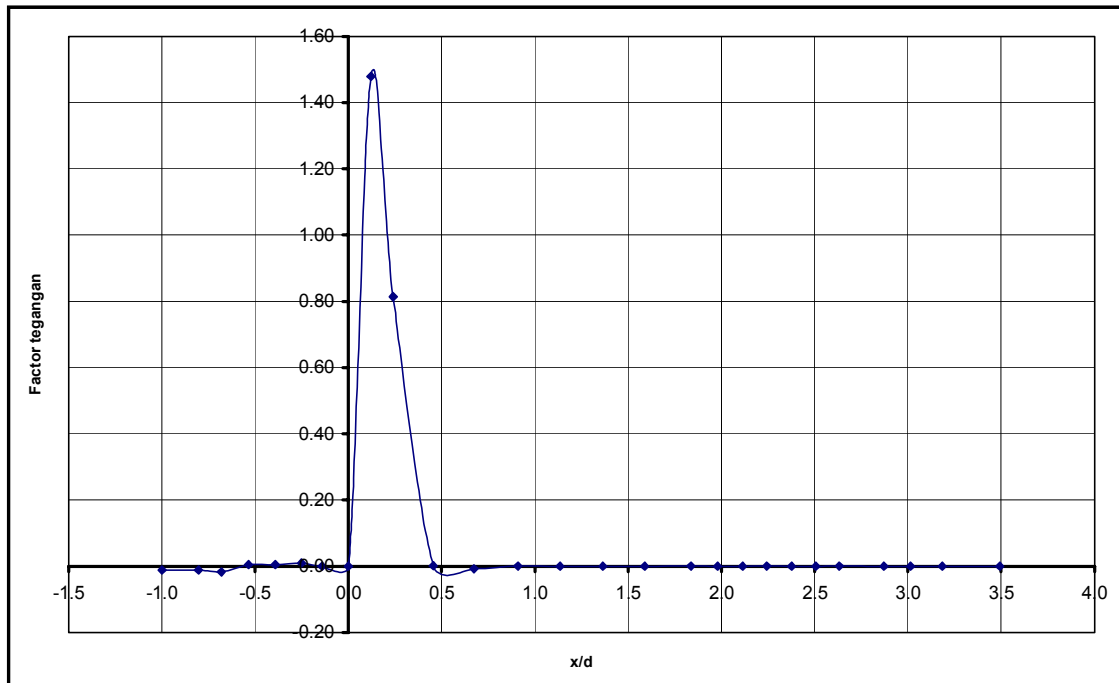 *s* = length of side of square cross section, mm

Figure 6 shear stress distributions in circular-to-square stepped-shaft due to twisting moment

Results & Discussion

Figure 6 shows the shear stress contour plot for the shaft considered. It can be seen that the maximum shear stress located at region of change in cross-section, exactly at the midside of the cross section with fillet. The maximum stresses were then recorded and divided by the nominal stress [Equation (5)] to obtained stress concentration factors. A graph of stress concentration factors can then be plotted as shown in Figure 7.
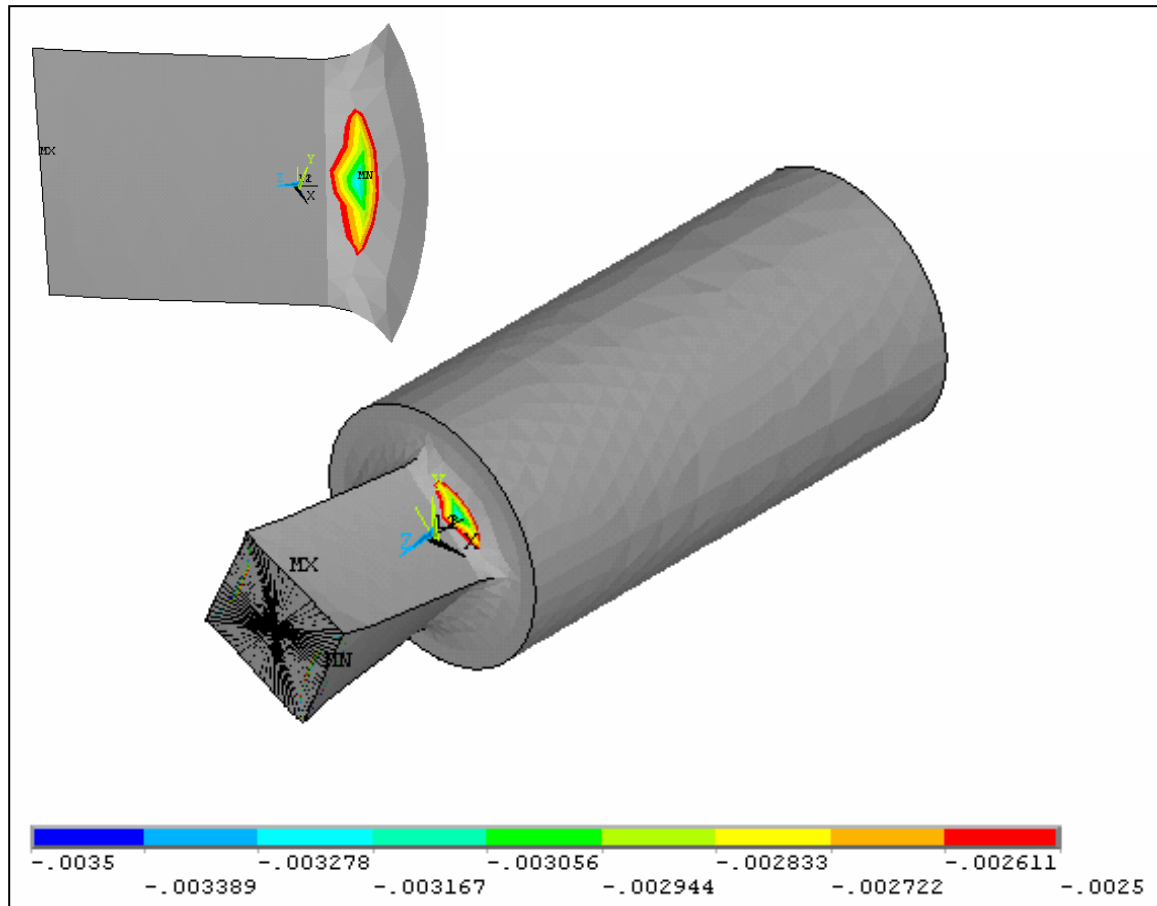

Figure 6 shear stress contour of circular-to-square stepped shaft due to pure torsion

As can be seen from Figure 7, stress concentration factors decrease as the radius of filled to diagonal ratio increase. The graph shows a similar trend to the well-known Peterson's graph for stress-concentration factor for stepped shaft of uniform circular of square cross-section. By comparing Figure 7 with Figure 3 and 4, it seems that the effect of diameter to diagonal ratio, *D/d*, is of less significant. It should be noted, however, that the spectrum of this ratio is small compared with Figure 3 and 4. A wide range of *D/d* values should be included and this is the subject of further publication elsewhere.

Conclusion

A graph for stress concentration factors of stepped shaft of circular to square cross section under pure bending load was produced. This graph was derived from parametric finite element study and can be directly used for design purpose. This graph shows a similar trend to the well known Peterson's graph for stress concentration factors of uniform circular or square stepped shaft.
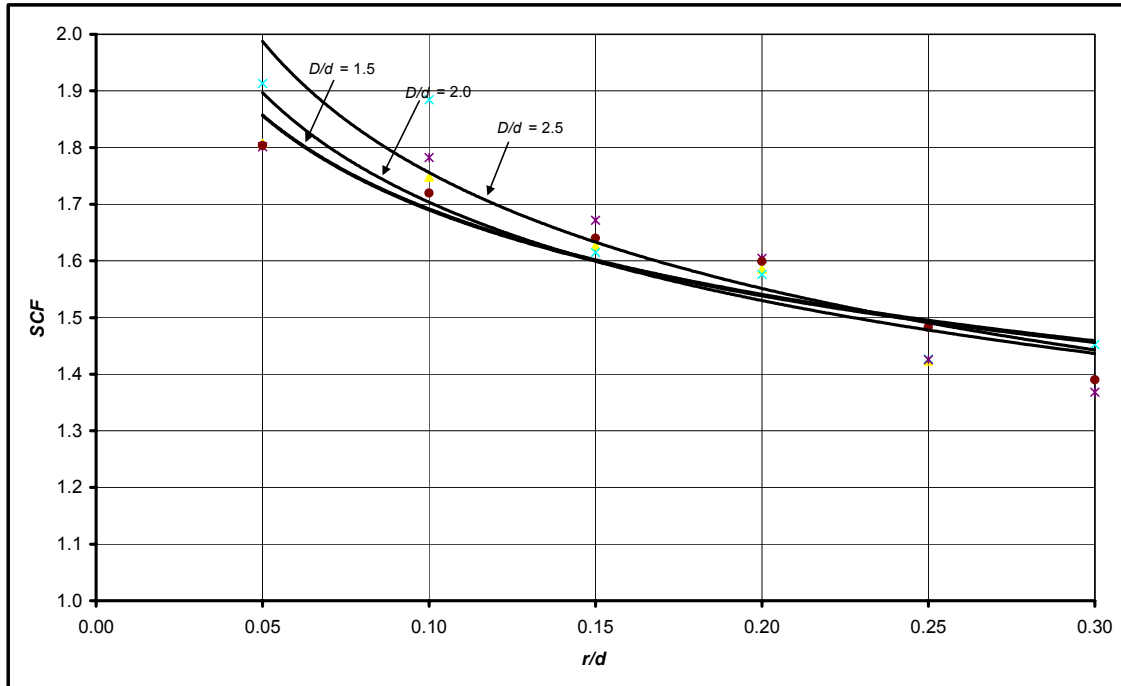
Figure 7 Stress concentration factor of stepped-shaft of circular-to-square cross section under torsion

Acknowledgement

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