Plastic Collapse Of Pipe Bends Under Combined External Pressure and In-Plane Bending

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Abstract

Pipe bend is a component of piping systems that often absorbs large loads and thermal expansions. These extreme loads could generate elastic-plastic cross-section deformation. In this paper, four load types are investigated. They are proportional loading, sequential pressure-moment loading, sequential moment-pressure loading, and twist loading. Theoretical analysis based on the Finite Element Method (FEM) for plastic collapse of pipe bends with attached to straight pipes is investigated. The analysis is carried out numerically covering small and large deformation of pipe bends. The categories of ductile failure loads are defined: limit load, plastic load, and instability loads. Different cases of application of loads are investigated such as pressure – only loading, bending – only loading, twist – only loading, combined loading limit loads, combined instability loads, and combined loading plastic loads. The plastic collapse criteria are used as a function of plastic load calculation. The objective measure of failure is given by plastic instability load. The effects of four types of load are shown in graphics.

Keywords: pipe bend; geometric non-linearity; plastic criterion; plastic instability; finite element limit analysis.

1. Introduction

The pipelines that transportation oil and natural gas in deep water are acted by different kinds of loading but for the load conditions and characteristic effect to be use in the design pipeline system are defined: sustained loads, occasional loads, and expansion loads. The sustained loads arising from the physical existence of the pipeline system, acting on the system under design conditions and include pressure, self-weight, fluid weight and insulation weight. Occasional loads arise from mechanical forces but are expected to occur during only small proportion of plant, such as intermittent operational loads or over load due to fault conditions. Expansion loads arise when the piping system experiences change in temperature over the operating cycle. Piping design always against with failure, seek to prevent failure due to the action of these loads. The codes guard against failure through appropriate choice of material and limiting the loads acting on the system. There are three types of failure that considered in routine design, such as gross plastic deformation, incremental plastic collapse, and fatigue. Gross plastic deformation is the fundamental ductile failure mode associated with static loading. It is prevented by restricting the magnitude of sustained and occasional loads. Incremental plastic collapse is a ductile failure mode associated with cyclic loading. It is prevented by limiting the magnitude of the static sustained loads plus the cyclic thermal expansion loads. Fatigue failure may occur at stress concentrations in the system after a finite number of load cycles, which in turn may determine the design life of the piping system. The flexibility is depends on the magnitude of thermal stresses resulting from cyclic loading of the system. The problems of incremental plastic collapse can be minimised by ensuring the system has sufficient flexibility to absorb thermal expansion without inducing excessive stress, deformation or connection forces in the system. System flexibility is enhanced by incorporating flexible components in the system. Pipe bends are commonly used components in a piping system. They are very flexible compared to the straight pipes, due to the complex deformation they exhibit under bending loads. When the pipe bend is subject to a bending load, the cross-section of pipe changes shape from a circle to an oval. Pipe bend normally reduces the reaction forces and moments within the piping system under thermal loading and it become easier to satisfy the stress limits. The deformation of the cross-section may enhance or reduce the strength and stiffness of the bend, depending on the direction of the moment. The behaviour of cross-section will become more complex again when the bends is pressurised, due to responses of coupling between the pressure and the bending. When a pressurised pipe bend is subject to a bending moment, the pressure acts against the ovalisation deformation. The first couple pressure-bending analysis is presented by Rodabaughand George [1] in 1957. Different studies had earlier been carried out to evaluate the limit loads of elbows. Marcal (1967) was the first to present the results for elastic-plastic behaviour of pipe bends with in-plane bending moment. Crandall and Dahl [2] showed that the relationship between pressure and ovalisation is non-linear, even for small deformation of the cross-section. Therefore, the small displacement theory of [1] does not describe the true nature of the pressure –bending effect, although it is included as an option in some piping design codes.

Table I **Nomenclature**

Symbol	Definition	Symbol	Definition
D	Diameter of pipe cross-section	P	Applied internal pressure
E	Elastic modulus	p	Normalised internal pressure
h	Pipe bend factor	R_b	Mean radius of elbow cross-section
L	Length of the straight pipe	r_m	Mean radius of elbow cross-section
M	Applied moment	t	Elbow wall thickness
m	Normalized moment	z	Moment multiplier
υ	Poison ratio	σ_y	Yield stress

2. Analysis

Figure 1 show the 90º pipe bend that consist of three piping systems such as comprise a 90º bend and two attached equal length straight pipe runs terminating at stiff flanges, considered in the present work. The mean radius and thickness of the pipe are denoted by *r* and *t*, respectively and the bend radius by *R*. The mean cross-sectional radius of the bend and straight pipe was set at r_m = 250 mm. Important non dimensional variables related to the bend geometry are the bend radius ratio R_b / r_m was fixed at 3 and for the bend radius to thickness ratio r_m / t and bend factor $h = \frac{R_b t}{r_m^2}$ were varied by changing the wall thickness, *t*. Those non dimensional variables were systematically varied to quantify the effect of pipe bend geometry on plastic limit load. Three values of thickness were considered: *t* = 15, 20, and 28 mm, giving pipe bend factors, *h* in the range 0.18 – 0.336. All parameters geometry of pipe bends are summarised in table II. One notable point of the present FE analysis is the geometry of the pipe. The piping system as mentioned above: comprises a 90º bend that is attached to a straight pipe of length *L*. Such an attached straight pipe remove numerical difficulties to apply loading boundary conditions to the FE model, particularly in-plane bending. For this case the length of the attached straight piping was chosen to ensure that the bend response was independent of the rigid flanges at the end of the runs. From analysis showed that a sensitive of this condition was met for three systems for a straight length *L* = 10 r_m. The material property values used were elastic modulus, *E* = 200 GN/m^2, yield stress, σ_y = 300MN/m^2, and poison ratio, υ = 0.3. An elastic-perfectly plastic material model was used in all the analyses.

Table II. Pipe bend geometry parameters

Mean radius, r_m (mm)	Thickness, t (mm)	r_m / t	R_b / r_m	Pipe bend factor, h
250	15	16.67	3	0.18
250	20	12.50	3	0.24
250	28	8.93	3	0.336

2.1 Finite element modelling

The systems were modelled in NASTRAN using plate element and modelled by full element mesh. A convergence study was performed to establish a suitable mesh density for the model. The finite element used in the study was described 15 elements along the straight run, 30 elements along the bend, and 48 elements around the pipe circumference as shown in Fig. 2. The flanges were simulated by elastic beam elements with elastic modulus an order magnitude greater than the pipe material. A web of radial beam element from the centre of the pipe end to the flange was included to allow the bending moment to be applied as a point load at the centre of the end. In FE analysis, internal pressure was applied as a distributed load to the inner surface of the FE model. The bend is subject to in-plane moment bending, twist, and pressure loading. The moment bending was applied as a point load to the node at the centre or the web of beams at the end of the straight, while the twist was applied as appoint load to the node at the tangent at the end of the straight. The system was assumed to be closed at the ends, such that the internal pressure gives rise to an axial thrust in the system. This was applied to the flange as an edge pressure, which remains normal to the end of the pipe during deformation. Four loading sequences were considered in the investigation. In the proportional loading, the internal pressure, moment bending, and twist are applied to the model simultaneously. In P-M loading, the internal pressure is applied to a predetermined value then held constant as the moment is applied. In M-P loading, the moment is applied to a predetermined value then held constant as the internal pressure is applied. Similarly, in P-T loading, the internal pressure is applied to a predetermined value then held constant as the twist is applied. In T-P loading, the twist is applied to a predetermined value then held constant as the internal pressure is applied.

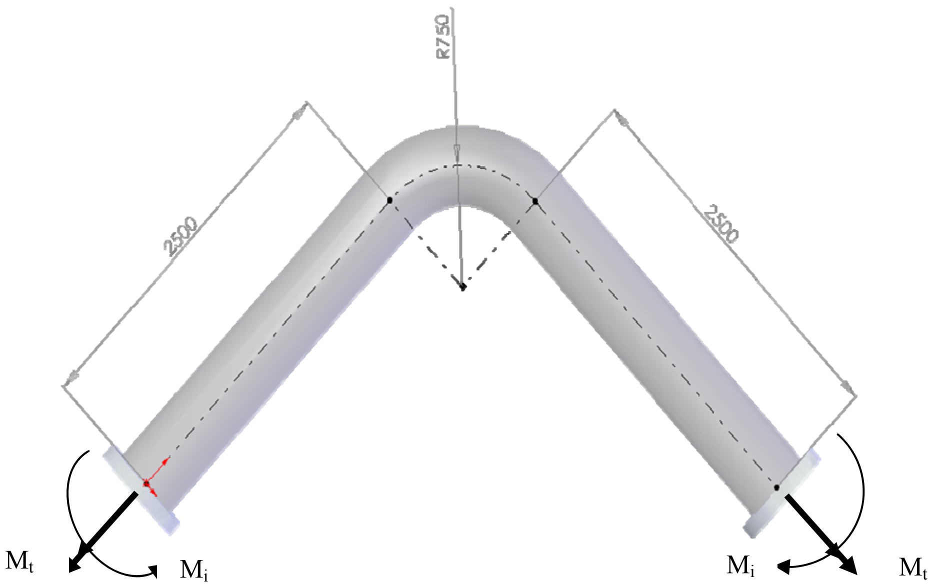


Fig. 1. Pipe bend attached to two straight runs subject to in-plane bending.

3. Results

The results of the analyses are obtained by applying the different cases of loads in term of graphic, such as pressure-only loading, bending-only loading, twist-only loading, combined loading limit loads, combined instability loads, and combined loading plastic loads. The internal pressure, P, is normalised with respect to the yield stress of a thin walled cylinder, such that the normalised pressure *p* is

$$p = \frac{\mathrm{P}r_m}{\sigma_y t}$$

Moment M is normalised with respect to the limit moment of straight pipe under pure bending, such that normal moment m is

$$m = \frac{M}{4r_m^2\sigma_y t}$$

3.1 Pressure – only, bending – only, and twist – only loading

Under pressure - only loading, first yield occurs in the middle of the bend at the inside surface of the intrados. As pressure is increased the plastic zone spread axially towards the junction with the straight run and circumferentially towards the extrados. The equivalent plastic strain distributions in the bend region at collapse for small and large deformation solution are shown in Fig. 3 for h = *0.24*. There is also some limited plastic redistribution in the circumferential direction but at failure, unstable or gross plastic deformation is restricted to a relatively small plastic zone around the intrados, as shown in Fig. 3a and b for small and large deformation analysis, respectively. The calculated limit pressure (small deformation collapse load) was close to that of straight pipe. The instability pressure (large deformation collapse load) was very close to the limit pressure. This indicates that large deformation effects are not significant in pressure-only loading. For the under in-plane moment bending, first yield occurs in the middle of the bend at the inside surface of the crown. As the load is increased beyond yield, the plastic zone spread both axially along the crown towards the straight run and circumferentially outwards, towards the extrados and the intrados. Fig.3 c and d show the deformation of the pipe bend which is applied moment only for small and large deformation analysis, respectively; it found that almost the entire bend experiences plastic deformation before failure occurred. The limit of the bend was significantly lower than the limit moment of a similar straight pipe. In the torsional moment (twist) the deformation similar to the internal pressure-only and in-plane bending –only loading. First yield occurs in the middle of the bend at the inside surface of the crown, and as the torsion moment is increased the plastic zone spread axially along the crown towards the straight run and circumferentially outwards, towards the extrados and the intrados, as shown in Fig . 3 e and f for small and large deformation analysis, respectively.

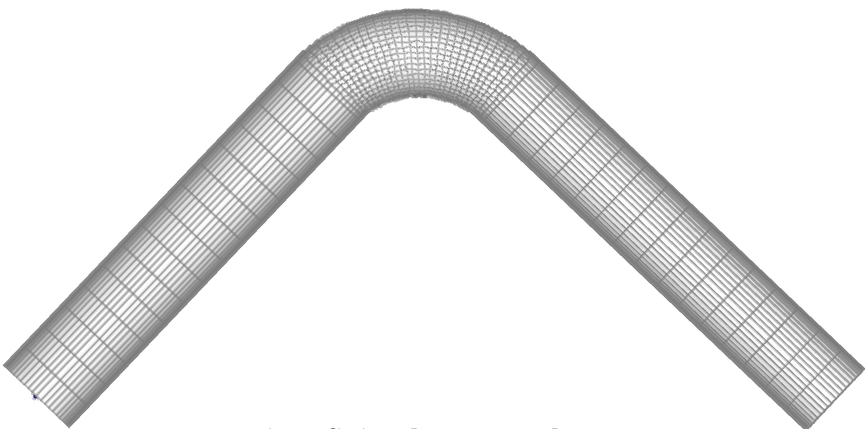
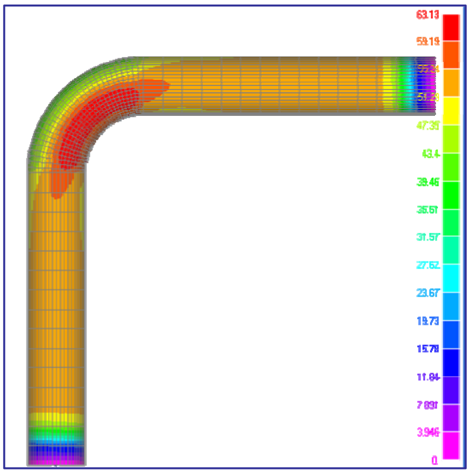
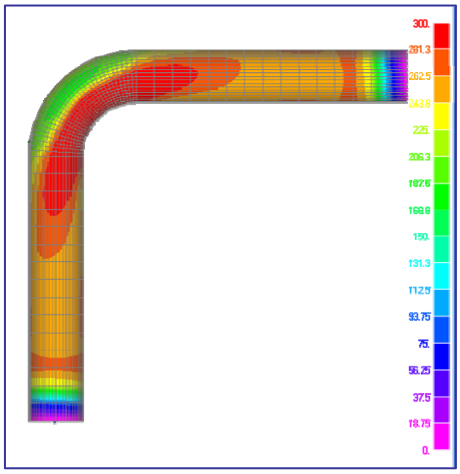


Fig. 2.finite element mesh



(a)



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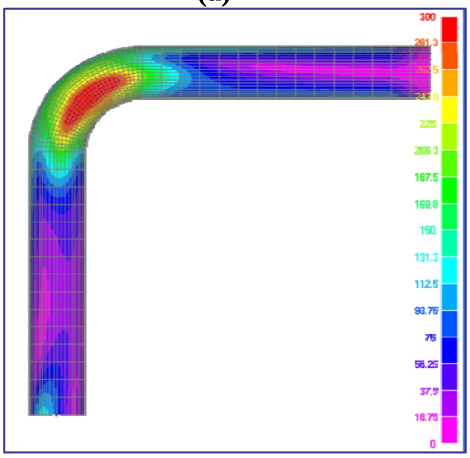

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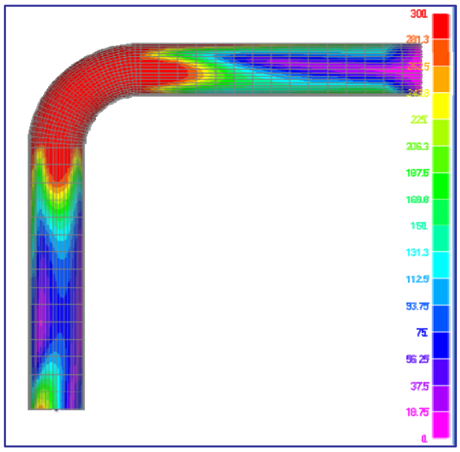

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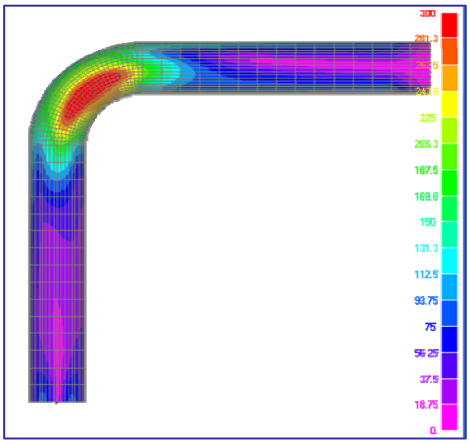
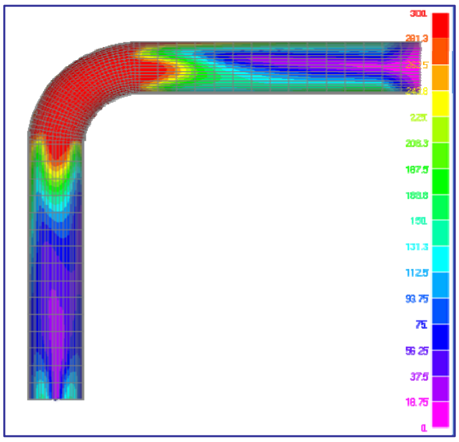


(e)



(f)

Fig. 3. von Mises equivalent plastic strain distribution at failure: (a) pressure only limit analysis, (b) pressure only large deformation analysis, (c) moment only limit analysis, (d) moment only large deformation analysis, (e) torsion only limit analysis, (f) torsion only large deformation analysis.

3.2 Combined loading limit loads

The limit load is path independent (independent of the loading sequence). This verified for the finite element model used in the investigation by calculating the limit load of the bend factor, *h = 0.24* for six load sequences. Fig.4 shown limit load surfaces obtained for proportional loading, P-M loading, M-P loading, P-M_t loading and M_t-P loading. The curves are almost coincident for most of the pressure range but there is different between the value of moment and twist and the others at high value of pressure and low value of moment and twist. In the normalised moment range 0-0.25, the pressure on the proportional loading and M-P limit surfaces exceed the limit pressure of the vessel. In the case of P-M loading, the maximum initial pressure that can be applied is, by definition, the limit pressure.

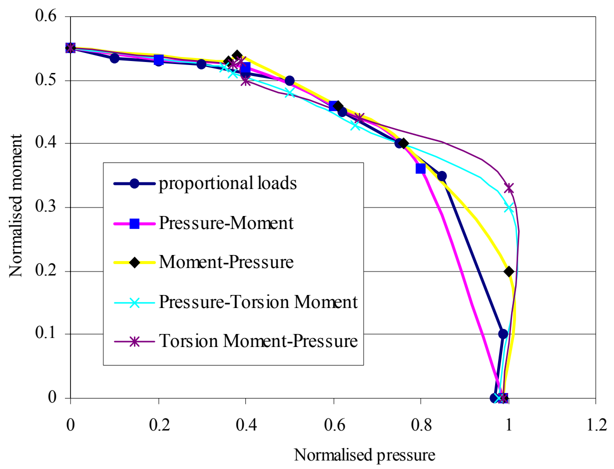


Fig. 4. Limit load surface for the h = 0.24 bend evaluated by proportional and sequential loading.

3.3 Combined loading instability loads

The plastic instability load surfaces are shown in Fig. 5 a-c for *h* = 0.18, 0.24, and 0.336 systems, respectively. Proportional loading, P-M loading, M-P loading, P-M_t loading, and M_t-P loading curves are compared with the limit load surface. Clearly, the order of loading significantly affects the calculated collapse load. The proportional loading, P-M loading, and P-M_t loading sequences give very similar failure surfaces. At low normalised pressures (less than 0.2), the ovalisation of cross-section lead to instability at loads below the limit load; that is, the structure exhibits geometric weakening. As the pressure increases, the ovalisation is countered by the internal pressure, which seeks to expand the cross-section as a uniform circle. At normalised pressure above 0.2, significant geometric strengthening is observed but reduces as the limit pressure is approached. The M-P loading sequence gives a distinctly different failure surface to the proportional and P-M load sequences. Under M-P load sequence, the initial bending moment cause the section to ovalise. Subsequent pressurisation counters the ovalisation until the cross-section become essentially circular, and as pressure increases, a failure mechanism similar to the pressure-only mechanism forms. The form of this mechanism is effectively independent of the initial bending load. The M_t-P loading, the initial torsion moment is applied and then held constant as pressure is applied. The initial torsion moment cause the pipe bend rotate and lead to changing the shape of cross-section.

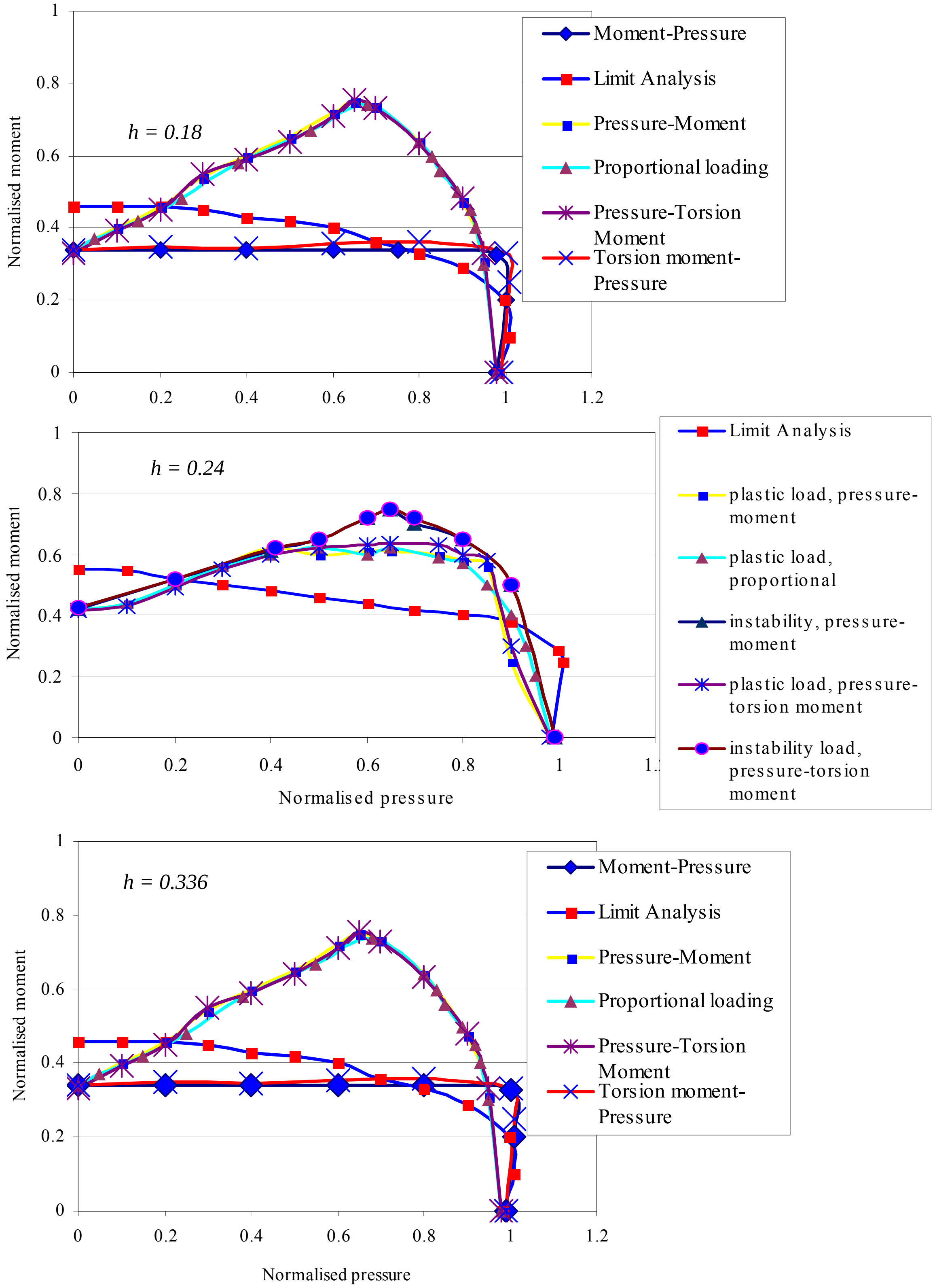
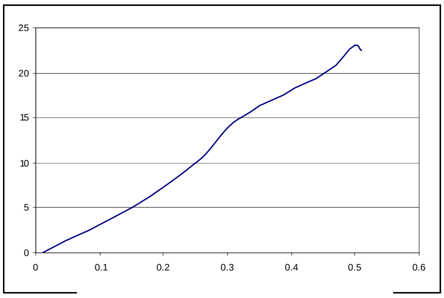


Fig.5 plastic instability load surfaces for the h = 0.18, b.) h = 0.24, c.) h = 0.336

3.4 Combined plastic loads

Plastic loads are defined by applying a specific criterion of plastic collapse characteristic load deformation curve obtained by plastic analysis. Following guidance from Gerdeen [3], the characteristic response curves for proportional and P-M loading were moment and end-rotation curves. For proportional loading, the pressure was initially applied to a constant value. This caused minor rotation of the flange. The moment was then applied and a moment-rotation curve plotted, taking the initial pressure induced rotation as a datum. In the M-P load sequence, an initial constant moment is applied and then pressure is increased until collapse occurs. In the P-M_t loading, first internal pressure is applied and the held constant as the torsion moment is applied. In the M_t-P loading, an initial constant torsion moment is applied and then pressure is increased until collapse occurs. The most significant load parameter in this case is probably pressure (the methodology is subjective) and, following by Gerdeen's recommendations, the appropriate deformation parameter are change in volume. Unfortunately, change in volume is not calculated in a conventional structural analysis and an alternative deformation is required. For M-P loading, it was decided to use end rotation as the deformation parameter. The initial moment was applied to a constant value. The pressure then was applied and a moment-rotation curve plotted, taking the initial moment induced rotation as datum. A typical pressure-rotation curve is shown in Fig. 6. The curve does not include the initial rotation due to application of the moment. In practice, the initial rotation may be significantly greater than the subsequent changes in rotation when the pressure is applied. Clearly, it is not possible to apply either the twice elastic slope or tangent intersection constructions to such a plot. For this reason, no plastic loads were calculated for M-P loading. The plastic load surfaces obtained by applying the tangent intersection method to the proportional loading and P-M loading curve for bend h = 0.24 are shown in Fig. 7. The two plastic curves are similar for low pressure but clearly differ as the normalised pressures exceed 0.8.

P, MPa



End rotation (Degrees)

Fig. 6. Typical pressure-rotation curve from moment-pressure large deformation analysis

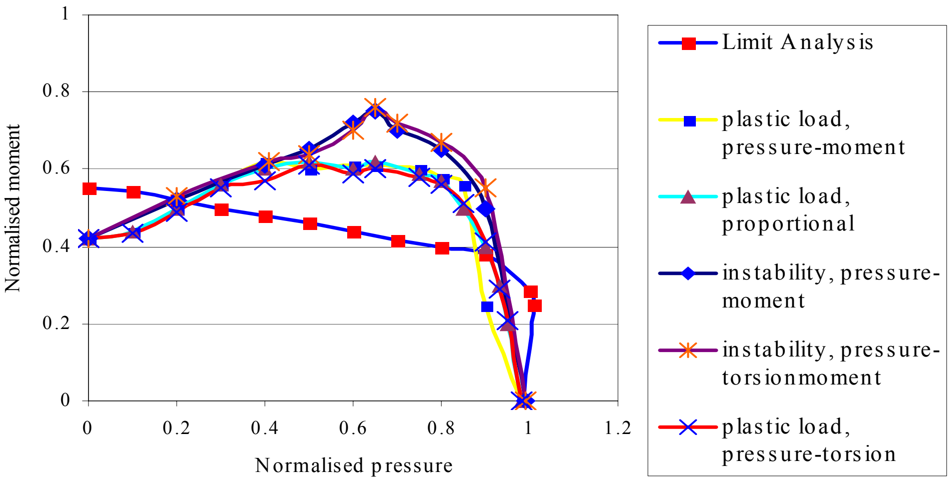


Fig. 7. Plastic loads for large deformation proportional and pressure moment loading. Limit and plastic instability loads shown for comparison

4. Conclusion

The results investigations show that geometric non-linearity is a significant consideration when calculating plastic failure load of pipe bends subject to combined loading. Significant geometric weakening is observed when the closing bending moment dominates. At higher pressure, both P-M loading and proportional loading cause considerable geometric strengthening. Calculating plastic load for the systems prove to be problematic. Plastic loads could not be defined for M-P loading when rotation was used as a deformation parameter, due to the general form of the characteristic response curve. Many different forms of characteristic curve were obtained for P-M loading, P-M_t loading and proportional loading. The twice elastic slope criterion could not be applied to the full range of configurations and plastic load was determined by applying the tangent intersection method. It was found that significant variation in calculated plastic pressure was possible, depending on how criterion was interpreted. The M-P, M_t-P, and proportional load cases gave similar plastic instability failure surfaces but when tangent intersection method was applied they gave distinctly different failure surfaces. This demonstrates that the calculated plastic load depends on the evolution of the failure mechanism rather than the actual state of collapse.

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